Reference 1: Functions

Definition of function

- Symbol: \( f : A \rightarrow B \)
- Spoken: “\( f \) is a function from \( A \) to \( B \)”, or “\( f \) maps \( A \) to \( B \)”,
- Usage: \( A \) and \( B \) are sets. Set \( A \) is called the domain and set \( B \) is called the codomain.
- Meaning: \( f \) is a machine that takes as input an element of set \( A \) and produces as output an element of set \( B \).
- Other notation: If an element \( a \in A \) is used as the input to the function \( f \), then the symbol \( f(a) \) is used to denote the corresponding output. We say that the output \( f(a) \in B \) is the image of the input \( a \in A \). In many cases, the sets \( A \) and \( B \) are sets of numbers. We will often use a machine diagram – a drawing like the one to the right – to visualize a function.

Displaying Numerical Functions on Graphs

The horizontal axis (often the “\( x \)-axis”) is used for the input numbers. For this reason, we will also refer to it as the input axis.

The vertical axis (often the “\( y \)-axis”) is used for the output numbers. For this reason, we will also refer to it as the output axis.

If an input \( a \in A \) causes an output \( f(a) \), then the point \((a, f(a))\) is on the graph, and vice-versa.

Interpreting Graphs as Functions

You’re all familiar with functions that are described by mathematical equations, and you’re used to the idea that those functions have corresponding graphs. (In Math 163A, we’ll be relating the behavior of these graphs to the behavior of the corresponding equations.) But now, we have a new definition of function, as a sort of “numerical machine”. Nothing in that definition requires that the function be described by an equation. In fact, sometimes a function will be described purely in words, or by a graph.

If a function is going to be described only by a graph (without a corresponding equation), then we need to be sure and describe clearly how that is going to work. That is, we need to describe how the graph is going to be used as a “numerical machine”.

To interpret a graph as a function:
- Given an input number \( a \), find the point on the graph whose left coordinate is the number \( a \).
- The point on the graph will have a number that is its right coordinate. That number is to be interpreted as the output number, \( f(a) \).