### Section 4-3: Derivatives of products and quotients

#### 4-3#10: $f(x) = 11x^3 \cdot 3^x$.
- **A** find $f''(x)$.
- **B** find $f''(0)$.
- **C** find $f''(1)$.

Review the Quotient Rule:

4-3#24: $f(x) = \frac{1-x^3}{1+x^2}$. Find $f'(x)$.

#### Class Drill 8: Don’t forget the easy derivative rules
(Topics from Section 4-3)

#### Quiz 9
Tuesday Feb 15th, 2011  (Day 25)

Review Product Rule with an example

\[ f(x) = 11x^3 \cdot 3^{(x)} \]

A) Find \( f'(x) \)

B) Find \( f'(0) \)

C) Find \( f'(1) \)

Solution

\[ f'(x) = \frac{d}{dx} 11x^3 \cdot 3^{(x)} \]

\[ = 11 \frac{d}{dx} x^3 \cdot 3^{(x)} \]

\[ = 11 \left( (3x^2) \cdot 3^{(x)} + x^3 \cdot (3^{(x)} \cdot \ln(3)) \right) \]

\[ = 11x^2 \cdot 3^{(x)} \left( 3 + x \ln(3) \right) \]
(B) \( f'(c) = 11 \cdot 0^2 \cdot 3^0 \cdot (3 + 0 \ln(3)) \)
\[ = 0 \]

(c) \( f'(1) = 11 \cdot 1^2 \cdot 3^1 \cdot (3 + 1 \ln(3)) \)
\[ = 33 \cdot (3 + \ln(3)) \]

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Review Quotient Rule with an example.

\[ f(x) = \frac{1-e^{x}}{1+e^{x}} \]
Find \( f'(x) \)

Solution \( f'(x) = \frac{\left( \frac{d(1-e^{x})}{dx},(1+e^{x}) \right) - (1-e^{x}) \left( \frac{d(1+e^{x})}{dx} \right)}{(1+e^{x})^2} \)
\[ = \frac{(-e^{x})(1+e^{x}) - (1-e^{x})(e^{x})}{(1+e^{x})^2} \]
\[
\frac{f(x)}{\ln e} = \frac{-e^x - e^x e^x - e^x + e^x e^x}{(1 + e^x)^2}
\]

\[
f'(x) = \frac{-2e^x}{(1 + e^x)^2}
\]

Do Class Drill 8:

Don't forget the Easy Derivative Rules

Products with one factor constant do not require product rule

Quotients with constant on bottom do not require quotient rule.
Class Drill 8: Don’t Forget the Easy Derivative Rules

[1] Let \( f(x) = 7(x^2 + 3x + 5) \).
(A) Find \( f'(x) \), using the Product Rule to deal with the 7 in front.

\[
\begin{align*}
  f'(x) &= \frac{d}{dx} \left( 7 \cdot (x^2 + 3x + 5) \right) \\
    &= (\frac{d}{dx} 7) \cdot (x^2 + 3x + 5) + 7 (\frac{d}{dx} (x^2 + 3x + 5)) \\
    &= 0 \cdot (x^2 + 3x + 5) + 7 (2x + 3) \\
    &= 14x + 21 = 14x + 21 \\
\end{align*}
\]

(B) Start over. Find \( f'(x) \) again, this time using the Constant Multiple Rule to deal with the 7 in front.

\[
\begin{align*}
  f'(x) &= \frac{d}{dx} 7 \cdot (x^2 + 3x + 5) \\
    &= 7 \frac{d}{dx} (x^2 + 3x + 5) \\
    &= 7 (2x + 3) = 14x + 21
\end{align*}
\]

[2] Let \( f(x) = \frac{x^2 + 3x + 5}{7} \).
(A) Find \( f'(x) \), using the Quotient Rule to deal with the fraction.

\[
\begin{align*}
  f'(x) &= \frac{(\frac{d}{dx}(x^2 + 3x + 5)) \cdot (7) - (x^2 + 3x + 5) \cdot (\frac{d}{dx} 7)}{7^2} \\
    &= \frac{(2x + 3) \cdot (7) - (x^2 + 3x + 5) \cdot (0)}{49} \\
    &= \frac{(2x + 3) \cdot 7}{49} = \frac{2x + 3}{7}
\end{align*}
\]

(B) Start over. Find \( f'(x) \) again, but this time do not use the Quotient Rule. Instead, start by rewriting \( f \) as a constant times a term in parentheses. Then use the Constant Multiple rule.

\[
\begin{align*}
  f(x) &= \frac{x^2 + 3x + 5}{7} = \left(\frac{1}{7}\right)(x^2 + 3x + 5) \\
  f'(x) &= \frac{d}{dx} \left(\frac{1}{7}\right)(x^2 + 3x + 5) \\
    &= \frac{1}{7} \left(\frac{d}{dx} x^2 + 3x + 5\right) = \frac{1}{7} \cdot (2x + 3) = \frac{2x + 3}{7}
\end{align*}
\]
[3] Let $f(x) = \frac{7}{e^{5x}}$.

(A) Find $f'(x)$, using the Quotient Rule to deal with the fraction. Simplify your answer.

$$f'(x) = \frac{(d\frac{7}{dx})(e^{5x}) - (7)(d\frac{e^{5x}}{dx})}{(e^{5x})^2}$$

$$= \frac{0(e^{5x}) - 7(5e^{5x})}{(e^{5x})^2}$$

$$= \frac{-35e^{5x}}{(e^{5x})^2} = \frac{-35}{e^{5x}}$$

(B) Start over. Find $f''(x)$ again, but this time do not use the Quotient Rule. Instead, start by rewriting $f$ as a constant times an exponential function with a negative sign in the exponent. Then use the Constant Multiple rule. Simplify your answer.

Rewrite $f(x) = \frac{7}{e^{5x}} = 7e^{-5x}$

$$f'(x) = \frac{d}{dx} 7e^{-5x}$$

$$= 7 \frac{d}{dx} e^{-5x}$$

$$= 7 \cdot (-5) e^{-5x}$$

$$= -35 \frac{e^{-5x}}{e^{5x}}$$

\[\text{de}^x = e^x\]