More topics from Section 3–5

Using the Sum & Constant Multiple Rule

<table>
<thead>
<tr>
<th>y' for $y = \frac{17x^6}{5} + \frac{17x}{5x} + \frac{17}{5x^5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>f'(x) for $y = \frac{2\sqrt{x}}{7} - \frac{3}{11x^{2/5}}$.</td>
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</tbody>
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Tangent line problems

underlying theory

Things that we know about the line tangent to the graph of $f$ at $x = a$.
- The point $(x, y) - (a, f(a))$ is on the tangent line. (It is the point of tangency.)
- The tangent line has slope $m = f'(a)$.

Therefore, the equation for the tangent line is $(y - f(a)) = f'(a)(x - a)$.

Tangent Line Problem

Let $f(x) = x^4 - 6x^2 + 10$.
(A) Find $f'(x)$
(B) Find slope of the line tangent to graph of $f$ at $x = -1$. Confirm with computer graph.
(C) Find equation of line tangent to graph of $f$ at $x = -1$.
(D) Find the value(s) of $x$ where the tangent line is horizontal. (Discuss the correct way of solving $4x^3 - 12 = 0$.)
Thursday, February 3, 2011 (Day 18)

Using the Sum & Constant Multiple Rule

\[ f(x) = \frac{17x^6}{5} + \frac{17x}{5} + \frac{17}{5x} + \frac{17}{5x^6} \]

Find \( f'(x) \)

Solution: Rewrite \( f(x) \) as constants times powers of \( x \).

\[ f(x) = \left( \frac{17}{5} \right) x^6 + \left( \frac{17}{5} \right) x + \left( \frac{17}{5} \right) x^{-1} + \left( \frac{17}{5} \right) x^{-6} \]

\[ = \left( \frac{17}{5} \right) \left( x^6 + x + x^{-1} + x^{-6} \right) \]
\[
\frac{d}{dx} \left( \frac{17}{5} \left( x^6 + x + x^{-1} + x^{-6} \right) \right) = \frac{17}{5} \left( 6x^5 + 1 + x^{-2} - 6x^{-7} \right)
\]
Example \[ f(x) = 2 \sqrt[5]{x} \]

Find \( f'(x) \)

Solution

Rewrite \( f(x) \)

First, rewrite \( f(x) \) as

\[ f(x) = (3/5)x^{3/5} - (3/7)x^{1/7} \]

Using the Constant Multiple Rule

\[ f'(x) = \frac{2}{5} x^{-2/5} - \frac{3}{7} x^{-6/7} \]
\[
\frac{3}{11} \left( \frac{2}{3} \right)^{\frac{2}{11}} 
+ \frac{6}{55} 
+ \frac{6}{55} \times 3^{\frac{2}{11}} 
\]

\[
\frac{2}{7} 
= \left( \frac{2}{7} \right)^{\frac{4}{7}} \times 3^{\frac{4}{7}} 
\]

\[
\frac{2}{35} 
= \left( \frac{2}{35} \right)^{\frac{4}{35}} 
\]
The line has slope \( m = 5 \).

Of tangency, tangent line, \( f(x) \) is the point

The point \( (x, y) = (a, f(a)) \) is on the

We know the things:

At the point where \( x = a \)

Is tangent to the graph of \( f(x) \)

What do we know about the line that

\text{Underlying Piece}

\text{Tangent Line Problems}
Get equation for tangent line.

In general, if you know that a line has slope \( m \) and passes through the point \( (x, y) = (a, b) \), then the "point-slope" form of the equation of the line is

\[
(y - b) = m(x - a)
\]

In our case, \( b = f(a) \) and \( m = f'(a) \)

\[
(y - f(a)) = f'(a)(x - a)
\]

Equation for the line tangent to graph of \( f \) at \( x = a \)
Tangent Line Problem

\( f(x) = x^2 - 6x^2 + 10 \)

(A) find \( f'(x) \)

\( f'(x) = \frac{d}{dx} (x^2 - 6x^2 + 10) \)

\( f'(x) = 2x - 12x \)

Solution:

\( f'(x) = 0 \)

\( x = \pm \sqrt{6} \)

\( y = 4x^2 - 12x \)
(B) Find the slope of the line tangent to graph of \( f \) at \( x = -1 \).

Solution: \[ m = f'( -1 ) \]
\[ = 4(-1)^3 - 12(-1) \]
\[ = 4(-1) + 12 \]
\[ m = 8 \]

(C) Find the equation of the line tangent to graph of \( f \) at \( x = -1 \).

Solution: We need to build this:
\[ (y - f(a)) = f'(a)(x-a) \]
Get parts for the equation.

\[ a = -1 \]

\[
\begin{align*}
f(a) &= f(-1) = (-1)^y - 6(-1)^2 + 10 \\
&= 1 - 6(1) + 10 \\
&= 5
\end{align*}
\]

\[
f'(a) = f'(-1) = 8 \quad \text{From part (B)}
\]

Substitute parts into the equation:

\[
(y - 5) = 8(x - (-1))
\]

Convert to slope-intercept form.
\[
y - 5 = 8(x + 1) \\
y - 5 = 8x + 8 \\
y = 8x + 13
\]

Equation of the tangent line

(D) At what \( x \)-values is the tangent line horizontal? That is, at what \( x \)-values is \( m = f'(x) = 0 \)?

Solution: Set \( f'(x) = 0 \) and solve for \( x \).

\[
4x^3 - 12x = 0 \\
x^3 - 3x = 0
\]
Solving $x^3 - 3x = 0$

The wrong solution method

Add $3x$ to both sides

$x^3 = 3x$

Divide by $x$ **Cannot do this if $x = 0$**

$x^2 = 3$

$x = \sqrt{3}$ or $x = -\sqrt{3}$

We missed the solution $x = 0$
Better solution: factor

\[ x^3 - 3x = 0 \]
\[ x(x^2 - 3) = 0 \]
\[ x(x + \sqrt{3})(x - \sqrt{3}) = 0 \]

Three solutions: \( x = 0 \), \( x = -\sqrt{3} \), \( x = \sqrt{3} \)