Day 15 is Friday, January 28, 2011

<table>
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<th>Section 3-4, do Class Drill 7</th>
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<td>Section 3-4 The Derivative, continued</td>
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<td>Rates of Change, continued</td>
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<td>Review example from yesterday involving</td>
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\[
f(x) = -x^2 + 7x = -x(x - 7)
\]

(A) Average rate of change from \( x = 1 \) to \( x = 5 \).
(Yesterday, the question was to find the slope of the secant line.)

(B) Find instantaneous velocity at \( x = 1 \) second

(C) Draw the graph of \( f \).

(D) The numbers that you got in (A) and (B) can be interpreted as slopes of lines that can be drawn on the graph of \( f \). Draw the lines.

<table>
<thead>
<tr>
<th>Physics terminology of position and velocity</th>
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<tr>
<td>Introduce terminology found on Reference 6</td>
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| Revisit previous example, rewording it in the terminology of a position function, average velocity, and instantaneous velocity |

<table>
<thead>
<tr>
<th>Definition of the Derivative</th>
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<tr>
<td>the definition. Project Reference 6</td>
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| do class drill 7a: Finding Derivatives graphically using a ruler |
Recall example of yesterday

\[ f(x) = -x^2 + 2x \]

A. Find the average rate of change from \( x = 1 \) to \( x = 5 \)

Solution \( m = \frac{f(5) - f(1)}{5 - 1} = \frac{10 - 6}{4} = \frac{4}{4} = 1 \)

Resume this example

B. Find the instantaneous rate of change of \( f \) at \( x = 1 \)

Solution We need to build this:

\[ M = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} \]
Four steps

Step 1: Find \( f(x) \)
\[ f(x) = -x^2 + 2x \]

Solution
So \( f(1) = -(1)^2 + 2(1) = 1 \)

Step 2: Find \( f(1+h) \)
\[ f(1+h) = -(1+h)^2 + 2(1+h) \]

Solution
So \( f(1+h) = -(1^2 + 2(1)(h) + h^2) + 2(1+h) \)
\[ = -1 - 2h - h^2 + 2 + 2h \]
\[ = 6 + 5h - h^2 \]
Step 3 find \( f(1+h) - f(1) \)

\[
f(1+h) - f(1) = (6 + 5h - h^2) - (x)
\]

\[= 5h - h^2\]

Step 4 find \( \frac{f(1+h) - f(1)}{h} \)

\[
\frac{f(1+h) - f(1)}{h} = \frac{5h - h^2}{h}
\]

\[= \frac{h(5-h)}{h}
\]

\[= 5-h\]
4

Step 5
Find $m$

Solution

$m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}

= \lim_{h \to 0} \frac{5 - h}{h}

= 5 - 0

= 5

$c$)

Draw the graph of $f$
Solution: \[ f(x) = x^2 + 7x = -x(x - 7) \]

- Parabola is facing down.
- X-intercepts at \((0, 0)\) and \((7, 0)\).
(D) The numbers that you got in (A) and (B) can be interpreted as slopes of lines that can be drawn on the graph of f. Draw the lines.

Solution

(A): $m = 1 = \text{average rate of change of } f \text{ from } x = 1 \text{ to } x = 5$

$= \text{slope of } \text{secant line that touches graph at } x = 1 \text{ and } x = 5$

(B): $m = 5 = \text{instantaneous rate of change of } f \text{ at } x = 1$

$= \text{slope of the line tangent to graph of } f \text{ at } x = 1$
Physics terminology of position and velocity

See reference 6 in course packet

Example involving that terminology

An object moves in 1 dimension.
Its position at time $x$ is given by the function $f(x) = -x^2 + 7x$ meters.

(A) Find the average velocity from time $x=1$ to time $x=5$.

Solution: Same as earlier problem:

Average velocity $= \frac{f(5) - f(1)}{5-1} = 1$

(B) Find the instantaneous velocity at time $x=1$.

Solution: Instantaneous velocity $= f'(1) = -2x + 7 = 5$
\[ M = \lim_{{h \to 0}} \frac{f(1+h) - f(1)}{h} = \ldots = 5 \]

Same as earlier problem

The Derivative
See Reference 6
Class Drill 7: Finding Derivatives Graphically Using a Ruler

The goal: Given the graph of \( f \) on the top axes on the next page, make a graph of \( f' \) on the bottom axes.

On the graph of \( f' \), the input will be \( x \) and the output will be \( f'(x) \). Remember the graphical interpretation of \( f'(x) \):

**Definition of the Derivative**
- **symbol:** \( f'(a) \)
- **graphical interpretation:** \( f'(a) \) is the number that is the slope of the line tangent to the graph of \( f \) at the point where \( x = a \).

Part 1: Prepare the data for your graph of \( f' \) by filling out the following table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>what to do on the graph of ( f )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2)</td>
<td>Draw the line tangent to the graph of ( f ) at the point where ( x = -2 ) and find its slope ( m ). This slope ( m ) will be the value of ( f'(-2) ).</td>
<td>( f'(-2) = -5 )</td>
</tr>
<tr>
<td>(-1)</td>
<td>Draw the line tangent to the graph of ( f ) at the point where ( x = -1 ) and find its slope ( m ). This slope ( m ) will be the value of ( f'(-1) ).</td>
<td>( f'(-1) = -2 )</td>
</tr>
<tr>
<td>( 0 )</td>
<td>Draw the line tangent to the graph of ( f ) at the point where ( x = 0 ) and find its slope ( m ). This slope ( m ) will be the value of ( f'(0) ).</td>
<td>( f'(0) = 0 )</td>
</tr>
<tr>
<td>( 1 )</td>
<td>Draw the line tangent to the graph of ( f ) at the point where ( x = 1 ) and find its slope ( m ). This slope ( m ) will be the value of ( f'(1) ).</td>
<td>( f'(1) = 2 )</td>
</tr>
<tr>
<td>( 2 )</td>
<td>Draw the line tangent to the graph of ( f ) at the point where ( x = 2 ) and find its slope ( m ). This slope ( m ) will be the value of ( f'(2) ).</td>
<td>( f'(2) = 4 )</td>
</tr>
<tr>
<td>( 3 )</td>
<td>Draw the line tangent to the graph of ( f ) at the point where ( x = 3 ) and find its slope ( m ). This slope ( m ) will be the value of ( f'(3) ).</td>
<td>( f'(3) = 4 )</td>
</tr>
<tr>
<td>( 4 )</td>
<td>Draw the line tangent to the graph of ( f ) at the point where ( x = 4 ) and find its slope ( m ). This slope ( m ) will be the value of ( f'(4) ).</td>
<td>( f'(4) = 6 )</td>
</tr>
</tbody>
</table>

Part 2 is on the next page.
Part 2: Using the \((x, f'(x))\) data from your table, make a graph of \(f'\).