Exam 1 Scores:

Average ≈ 66% (low C+)

A: 25
A-: 6
B+: 13
B: 6
B-: 9
C+: 2
C: 16
C-: 5
D: 20
F: 16

hole in the middle
Section 3-1 Limits

The definition of limit

**Symbol:** \[ \lim_{x \to c} f(x) = L \]

Spoken: "the limit, as \( x \) approaches \( c \), of \( f(x) \), is \( L \)."

**Less-abbreviated symbol:** \( f(x) \to L \) as \( x \to c \).

Spoken: "\( f(x) \) approaches \( L \) as \( x \) approaches \( c \)."

**Usage:** \( x \) is a variable.
\( f \) is a function
\( c \) is a real number constant
\( L \) is a real number constant

**Meaning:** As \( x \) gets closer & closer to \( c \), but not equal to \( c \),
the value of \( f(x) \) gets closer & closer to \( L \) (and may actually equal \( L \))"
<table>
<thead>
<tr>
<th>( f )</th>
<th>( g )</th>
<th>( h )</th>
<th>( i )</th>
<th>( j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (9) )</td>
<td>( (10) )</td>
<td>( (11) )</td>
<td>( (12) )</td>
<td>( (13) )</td>
</tr>
<tr>
<td>( \lim_{x \to a} f(x) = L )</td>
<td>( \lim_{x \to a} g(x) = L )</td>
<td>( \lim_{x \to a} h(x) = L )</td>
<td>( \lim_{x \to a} i(x) = L )</td>
<td>( \lim_{x \to a} j(x) = L )</td>
</tr>
</tbody>
</table>

Use the graph to fill in the table (Extra copies of the graph are on back).
So far: graphical approach to limits

- Class drill 4 involved graph \( \Rightarrow \) descriptions of the limits.

- Now do another type of graphical problem.

  \[ \text{description of limits} \quad \Rightarrow \quad \text{graph}. \]

Exercise 3.1440 Sketch a graph that satisfies all three conditions:

- \( f(1) = 1 \)

- \( \lim_{{x \to 1^-}} f(x) = 2 \)

- \( \lim_{{x \to 1^+}} f(x) = -3 \)
Solution

Three locations are important:

\[(x, y) = (1, 1)\]
\[(x, y) = (1, 2)\]
\[(x, y) = (1, -3)\]

Plot open circles at those locations.

As \(x\) gets close to 1 from the left, \(y\) gets close to 2.

Because

\[
\lim_{{x \to 1^-}} f(x) = 2
\]

\((1, 1)\) filled in because \(f(1) = 1\)

\((1, -3)\)
Example #2 3-1 #42

Sketch a possible graph of a function \( f \) that satisfies all three conditions:

- \( f(-5) = 3 \)
- \( \lim_{x \to -5^-} f(x) = 4 \)
- \( \lim_{x \to -5^+} f(x) = 4 \)

Solution: important locations: \((x,y) = (-5,3)\) and \((x,y) = (-5,4)\)