Tuesday Jan 11, 2011

Section 2-5 Exponential Functions

Compare these two functions

\[ f(x) = x^2 \quad \text{Power function} \]
\[ g(x) = 2^x \quad \text{Exponential Function} \]

\[ \text{Variable base} \quad \text{Constant exponent} \]
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In general, an Exponential Function is one of the form \( g(x) = b^x \) where \( b \) is a real number with \( 0 < b < 1 \) or \( 1 < b \).
\[
\begin{align*}
&\begin{array}{c}
e^2 \quad e^3 \quad 1 \\
\end{array} = \\
&\begin{array}{ccc}
-1 & 0 & 1 \\
-1 & -e^2 & -e^2 \\
\end{array}
\end{align*}
\]
Graph a couple of exponential functions

Make a table of x, y values

\[
\begin{array}{c|ccc}
  x & 2^x & e^x & 3^x \\
\hline
-3 & \frac{1}{8} & \frac{1}{e^3} & \frac{1}{27} \\
-2 & \frac{1}{4} & \frac{1}{e^2} & \frac{1}{9} \\
-1 & \frac{1}{2} & \frac{1}{e} & \frac{1}{3} \\
0 & 1 & 1 & 1 \\
1 & 2 & e & 3 \\
2 & 4 & e^2 & \frac{9}{2} \\
3 & 8 & e^3 & 27 \\
\end{array}
\]

all bases are $> 1$

horizontal asymptote on left at $y = 0$
Introduce a number $e$ such that $2 \leq e \leq 3$.

Fill in the empty column of table with values of $e(x)$ and graph.

New graph

$y = 3^x + 2$

Use transformations

Basic graph $y = 3^x$  

$(0, 1)$  

$(1, 3)$

Horizontal asymptote at $y = 0$
2nd graph \( y = 3^x + 2 \)

Additive constant \( c = 2 \)
on outside
So add \( c = 2 \) to all \( y \)-values.
Graph moves up 2 units.

\[
\frac{1}{3} + 2 = \frac{7}{3}
\]

\[ \text{Evil notation} \]
Another example

Graph: $y = 3^{(-x)}$

1st graph: $y = 3^x$

Solution: Use transformations. $y = 3^x$ becomes $y = 3^{(-x)}$

$C = -1$ - Multiply constant on the inside.

$D = 1$ - Divide $x$-values by $C = -1$. (same thing as multiplying.)

1st point: $(1, 3)$
2nd point: $(0, 1)$
3rd point: $(-1, 3)$
4th point: $(0, 1)$
Notice: $3^{-x}$ can be rewritten as

$$y = 3^{(-x)} = \frac{1}{3^{(x)}} = \left(\frac{1}{3}\right)^x$$

Exponential function with base $b = \frac{1}{3}$

Notice: $0 < \frac{1}{3} < 1$