Class Drill 14: Using the Graphing Strategy to Graph a Rational Function

The goal is to graph the function \( f(x) = \frac{x + 3}{x - 2} \).

Step 1. Analyze \( f(x) \).
- Find the \( y \)-intercept and the \( x \)-intercept.
- Are there any vertical asymptotes? If so, what are their line equations?
- Are there any horizontal asymptotes? If so, what are their line equations?
- Make a sign chart for \( f \) and use it to determine where \( f \) is positive, negative, or zero.

Step 2. Analyze \( f'(x) \).
- Find \( f'(x) \), factor it, and then find the partition numbers for \( f'(x) \).
- Construct a sign chart for \( f'(x) \) and use it to determine the intervals on which \( f \) is increasing and decreasing.
- There will be no local maxima or minima. Why not?
Step 3. Analyze $f''(x)$.
- Find $f''(x)$, factor it, and then find the partition numbers for $f''(x)$.
- Construct a sign chart for $f''(x)$ and use it to determine the intervals on which $f$ is concave up and concave down.
- There will be no inflection points. Why not?

Step 4: Sketch the graph of $f$.
- Plot the axis intercepts and label them with their $(x,y)$ coordinates.
- Draw the asymptotes and label them with their line equations.
- Using the other information from steps 1, 2, and 3, draw the graph.