Math 330B Homework Set #4, Due Tuesday, April 27, 2010

[0] Read Sections 5.1 of the textbook.

[1] (textbook problem 5.1 # 4ab) When the image of a figure under a reflection in a line is the figure itself, the line is referred to as a line of symmetry for the figure. For example, a rectangle has two lines of symmetry: the perpendicular bisectors of the two pairs of opposite sides.

(a) For each of the following figures
- draw the figure
- draw all lines of symmetry for the figure (If there are none, indicate this.)
- Describe each line of symmetry. (Use colors in your drawings and descriptions.)
  i. Segment
  ii. Line
  iii. Square
  iv. Rhombus
  v. Equilateral Triangle
  vi. Trapezoid
  vii. Circle
  viii. Kite
  ix. a figure of your own design. (To get credit, it must be unique!!)
  x. a figure of your own design. (To get credit, it must be unique!!)

[2] (textbook problem 5.1 # 4c) Describe all the lines of symmetry of a regular \( n \)-gon.

[3] (textbook problem 5.1 # 8) Prove that any isometry of the plane is also a transformation of the plane. That is, prove that

For any function \( f : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \), if \( f \) preserves distance then \( f \) is one-to-one and onto.
[4] (textbook problem 5.1 # 11) Classify each of the following statements as true or false. If a statement is true, prove it. If it is false, give a counterexample
(a) For all points A, B, C, the composition $\tau_{AB} \circ \tau_{BC}$ is a translation.
(b) For all points A, B, C, D, the composition $\tau_{AB} \circ \tau_{CD}$ is a translation.
(c) For all points A, B, C, D, the composition $\tau_{AB} \circ \tau_{CD} = \tau_{CD} \circ \tau_{AB}$.

[5] (textbook problem 5.1 # 15) In Now Solve This 5.5, we pointed out that when the point $(x,y)$ is reflected in the x-axis in a Cartesian coordinate system, its image is $(x,-y)$.
If $M_x$ denotes the reflection in the x-axis, we say that $M_x$ maps $(x,y)$ to $(x,-y)$, and we write $M_x(x,y) = (x,-y)$. Find the image of $(x,y)$ under each of the following isometries.
   a) Reflection in the y-axis.
   b) Reflection about the line $y = x$.
   c) A half-turn about the origin.
   d) $R_{(0,0), \pi/2}$
   e) $\tau_{AO}$, where A is the point $(a,0)$ and O is the origin.
   f) $\tau_{OB}$, where B is the point $(0,b)$ and O is the origin.
   g) $\tau_{OC}$, where C is the point $(a,b)$ and O is the origin.

[6] (textbook problem 5.1 # 16) Consider the mapping $F$ defined for all points $(x,y)$ in the plane as follows:
   If $y \geq 0$, then $f(x,y) = (x,y+1)$
   If $y < 0$, then $f(x,y) = (x,y-1)$
   a) Find the domain and range of $F$.
   b) Is $F$ one-to-one?
   c) Is $F$ a transformation?
   d) Is $F$ an isometry?

[7] (textbook problem 5.1 # 17) Which of the following mappings $F$ from the plane to the plane are transformations? Which are isometries?
   a) $F(x,y) = (x,y+1)$
   b) $F(x,y) = (x-1,y+1)$
   c) $F(x,y) = (|x|,|y|)$
   d) $F(x,y) = (x^2, y^2)$
   e) $F(x,y) = (1/x,1/y)$