Math 330B Homework Set #2, Due Friday, April 9, 2010

[0] Read Sections 4.2 and 4.3. Print this page to use as a cover sheet.

[1] 4.2 #2
• typo: Must assume AB = BC.
• Apply Theorem 4.7 to $\triangle BDA$ and also to $\triangle BDC$. You will end up with two equations involving ratios. Combine them into one equation. Then use Theorem 4.3.

[2] 4.2 #3
• Proof that AH = HM. Using this fact, you should be able to prove what you need to prove about AB/AH.
• Then Apply Theorem 4.7 to $\triangle BCH$. You should be able to recognize the resulting ratio as being that of a famous triangle.

[3] 4.2 #7
• Proof that AH = HM. Using this fact, you should be able to prove what you need to prove about AB/AH.
• Then Apply Theorem 4.7 to $\triangle BCH$. You should be able to recognize the resulting ratio as being that of a famous triangle.

[4] 4.2 #14
• typo: problem should read $(AT)^2 = AB \cdot AC$.
• Proof by contrapositive. That is, prove that if line(AT) is not tangent to the circle, then $(AT)^2$ does not equal $(AB)(AC)$. You will need to use an early theorem in the book that says that a tangent to a circle touches the circle exactly once. (Find that theorem.) So in other words, if line(AT) is not tangent to the circle, then line(AT) must touch the circle at an additional point besides point T.
• You will need to use Theorem 4.12.

[5] 4.2 #15
• typo: Prove that $A = abc/(4R)$
• Hint: Let D be the point on side BC such that AD is perpendicular to BC. So AD is the altitude from vertex A. Let $h = AD$. Then the area of $\triangle ABC$ will be $Area = (1/2)ah$.
• Now you need to somehow get $h$ out of this equation and get $b, c$, and $R$ into it. The trick is to find a triangle that involves a radial segment and also side AC and that is similar to $\triangle ABD$. Let point O be the center of the circle, and let point E be the point where line(AO) intersects the circle. That is, point E is the other end of the diameter segment drawn from point A. Now consider the angles of $\triangle AEC$. Note that $\angle ACE$ is a right angle (why?). And note that $\angle AEC \cong \angle ABD$ (why?). Therefore, $\triangle AEC \sim \triangle ABD$. (why?)
• Note that \( \frac{AE}{AB} = \frac{AC}{AD} \) (why?). That is, \( \frac{h}{c} = \frac{b}{2R} \). This equation can be used to eliminate the \( h \) in your equation for the area.

[6] 4.2 # 16
Show that point B lies on line(AC). To do this,
• Suppose that line(AC) intersects the left circle at point B1 and intersects the right circle at point B2. Then show that points B1 and B2 are actually the same point.
• Apply Corollary 4.1 to left circle to get an equation involving AC, B1C, and CD.
• Apply Corollary 4.1 to right circle to get an equation involving AC, B2C, and CE.
• Apply Theorem 2.6 to segments CD and CE.
• Do algebra to get the equation \( B1C = B2C \). Draw a conclusion from this equation.

[7] 4.3 # 1
• Let \( x_1 = \) length of base of \( \Delta_1 \), let \( x_2 = \) length of base of \( \Delta_2 \), let \( x_3 = \) length of base of \( \Delta_3 \). Let \( S = \text{area}(\Delta ABC) \).
• Using Theorem 4.14, get equation involving the ratios \( \frac{S_1}{S} \) and \( \frac{x_1}{AC} \). Similarly for the other three triangles.
• For the main computation, get an equation expressing the length AC in terms of \( x_1, x_2, x_3 \).
• Divide both sides of this equation by AC. The right side of this equation should now be a bunch of ratios of lengths.
• Replace these expressions with square roots of ratios of areas.
• Finally, multiply both sides of this equation by \( \sqrt{S} \).

[8] 4.3 # 5
• Consider the original hexagon, called hex1. The sides of this hexagon have length \( a \). But this hexagon can be divided up into six equilateral triangles, each of which has three sides of length \( a \). This tells us that in hex1, the distance from the center to a vertex is \( a \).
• Now consider one of the six triangles that makes up hex1. Find the height \( h \) of one of these triangles, in terms of \( a \). Notice that the foot of the altitude will be at the midpoint of the base. This midpoint will be one of the vertices of hex2. In other words, the height \( h \) is actually the distance from the center to a vertex of hex2. So the height \( h \) is also the length of one of the sides of hex2.
• From the known lengths of the sides of hex1 and hex2, you should be able to determine the ratio \( \frac{\text{area}(\text{hex}_1)}{\text{area}(\text{hex}_2)} \).
• Imagine repeating this process until you get to hex6.