Definition of Average Rate of Change
- **words:** the average rate of change of \( f \) as the input changes from \( a \) to \( b \)
- **usage:** \( f \) is a function that is continuous on the interval \([a,b]\).
- **meaning:** the number \( m = \frac{f(b) - f(a)}{b - a} \)
- **graphical interpretation:** \( m \) is the slope of the secant line containing \((a, f(a))\) and \((b, f(b))\).
- **remark:** The average rate of change \( m \) is a number.

Definition of Instantaneous Rate of Change
- **words:** the instantaneous rate of change of \( f \) at \( a \).
- **alternate words:** the derivative of \( f \) at \( a \).
- **symbol:** \( f'(a) \)
- **meaning:** the number \( m = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \)
- **graphical interpretation:** \( m \) is the slope of the line tangent to the graph of \( f \) at the point \((x, y) = (a, f(a))\).
- **remark:** The instantaneous rate of change \( f'(a) \) is a number.

Definition of the Derivative
- **words:** the derivative of \( f \)
- **symbol:** \( f' \)
- **meaning:** \( f' \) is a function. To describe a function, one must specify how it produces output for a given input. For input \( x \), the output \( f'(x) \) is the number \( f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \).
- **graphical interpretation:** For an input \( x \), the output \( f'(x) \) is the number that is the slope of the line tangent to the graph of \( f \) at the point \((a, f(a))\)
- **remark:** The derivative \( f' \) is a function.

Terminology of Position and Velocity
- **Time:** When our book uses mathematical functions to describe the motion of objects, \( x \) is a variable that represents the elapsed time.
- **Position:** To say an object is “moving in 1 dimension” means that it can go forward or backward in one direction but cannot turn. In such situations, a single coordinate can be used to keep track of the position of the object. A function called the *position function* gives the value of the coordinate at a given time. In our book, the position function is called \( f \). That is, at time \( x \), the coordinate of the object is the number \( f(x) \).
- **average velocity:** The words “the average velocity from time \( x = a \) to time \( x=b \)” mean the same thing as the words “the average rate of change of position from time \( t=a \) to time \( t=b \)”.
- **instantaneous velocity:** The words “instantaneous velocity at time \( x =a \)” mean the same thing as “instantaneous rate of change of position at time \( x = a \)” That is, the number \( f'(a) \).
- **velocity:** The word “velocity” means the same thing as the words “derivative of the position function”. That is, the velocity is the function \( f' \).