Class Drill 13: Using the Graphing Strategy to Graph a Rational Function

The goal is to graph the function \( f(x) = \frac{x + 3}{x - 2} \).

Step 1. Analyze \( f(x) \).
- Find the y-intercept.
- Find the x-intercepts.
- Are there any vertical asymptotes? If so, what are their line equations?
- Determine the end-behavior.
- Are there any horizontal asymptotes? If so, what are their line equations?
- Make a sign chart for \( f \) and use it to determine where \( f \) is positive, negative, or zero.

Step 2. Analyze \( f'(x) \).
- Find \( f'(x) \) and factor it.
- Find the partition numbers for \( f'(x) \).
- Construct a sign chart for \( f'(x) \) and use it to determine the intervals on which \( f \) is increasing and decreasing.
- There will be no local maxima or minima. Why not?
Step 3. Analyze \( f''(x) \).
- Find \( f''(x) \).
- Find the partition numbers for \( f''(x) \).
- Construct a sign chart for \( f''(x) \) and use it to determine the intervals on which \( f \) is concave up and concave down.
- There will be no inflection points. Why not?

Step 4: Sketch the graph of \( f \).
- Plot the axis intercepts and label them with their (x,y) coordinates.
- Draw the asymptotes and label them with their line equations.
- Using the other information from steps 1, 2, and 3, draw the graph.