Problem: 1 2 3 4 5 6 7 8 Total Rescaled
Score: 
Possible: 5 5 5 5 5 5 5 5 40 20

Math 330A Homework Set #8, Due Friday, March 12, 2010

[0] Read Section 3.2 of the textbook. Print this page to use as a cover sheet.

Problems [1] through [4] are proofs of the Pythagorean Theorem

[1] (based on textbook problem 3.1#9) Write a proof of the Pythagorean Theorem that uses these two pictures as an illustration.

[2] (based on textbook problem 3.1#1) The goal is to write a proof of the Pythagorean Theorem that uses the picture below as an illustration. In the figure, \( \triangle ABC \) is a right triangle with sides \( BC = a \), \( CA = b \), and \( AB = c \). Four triangles congruent to \( \triangle ABC \) have been assembled to form a rhombus of side length \( c \).

(a) Show that the outer figure with side \( c \) (the rhombus) is actually a square.
(b) Show that the inner figure is also a square. (It is clearly a rectangle, because it is a quadrilateral with four right angles. Your job is to show that the four sides have equal length.)
(c) Compute the area of the outer square in two different ways. Obtain the Pythagorean Theorem.
[3] (based on textbook problem 3.2#11) (Leonardo da Vinci’s Proof) Write a proof of the Pythagorean Theorem that uses the picture below as an illustration. In the figure, $\Delta ABC$ is a right triangle with sides $BC = a$, $CA = b$, and $AB = c$. Squares have been constructed on the three sides of $\Delta ABC$. Point $C'$ has been constructed so that $C'D = CA = a$ and $C'E = CB = b$.

(a) Show that $\quad \text{area}(CC'E'A) \cong \text{area}(C'CBD)$.
(b) Show that $\quad \text{area}(CC'E'A) \cong \text{area}(FIBA)$.
(c) Show that $\quad \text{area}(CC'E'A) \cong \text{area}(FIHG)$.
(d) Show that $\quad \text{area}(poly(CAEC'DB)) = \text{area}(poly(FABIHG))$.
(e) Show that the three shaded triangles have equal area.
(f) Use (d),(e) to obtain the Pythagorean Theorem.

**Background:** You may use this theorem:

*Theorem (SASAS congruence for quadrilaterals)* If two quadrilaterals have SASAS parts congruent, then the quadrilaterals are congruent.

[4] (based on textbook problem 3.2#10) The goal is to write a proof of the Pythagorean Theorem that uses the picture below as an illustration. In the figure, $\Delta ABC$ is a right triangle with sides $BC = a$, $CA = b$, and $AB = c$. Squares have been constructed on the three sides of $\Delta ABC$. Point $C'$ has been constructed so that $C'B' = CB = a$ and $C'A' = CA = b$.

(a) Show that $\quad \text{area}(ABB'A) = \text{area}(ACC'A') + \text{area}(CBB'C')$.

Show that $\text{area}(CBB'C') = a^2$. To do that, let $D$ be the point where line($A'C'$) intersects line($BC$), and do steps (b),(c),(d)

(b) Show that $\Delta CC'D \cong \Delta ABC$.
(c) Show that $C'D = a$.
(d) Show that $\text{area}(CBB'C') = a^2$.

Show that $\text{area}(ACC'A') = b^2$. To do that, let $E$ be the point where line($B'C'$) intersects line($A'C$), and do steps (e), (f), (g).

(e) Show that $\Delta CC'E \cong \Delta ABC$.
(f) Show that $C'E = b$.
(g) Show that $\text{area}(ACC'A') = b^2$.

(h) (Wrap-up) Use (a), (d), (g) to prove the Pythagorean Theorem.


(a) Use the Pythagorean Theorem to find $a$ in terms of $c$. Show all details.

(b) Use the Pythagorean Theorem to find $b$. Show all details.

(c) Use the Pythagorean Theorem to find $c$. Show all details.

(d) Using Theorems from Chapter 1, prove that $a = c/2$.

Hints
- Show that there exists $D$ such that $B - C - D$ and $CD = CB$.
- Consider triangles $\triangle ABC$, $\triangle ADC$, and $\triangle ABD$.

(e) Using (d) and Pythagorean Theorem, show $b = \frac{c\sqrt{3}}{2}$.

[6] (based on textbook problem 3.2#6)

(a) In the figure, Line$(A, P)$ is tangent to $Circle(O, OP)$. Express $AP$ in terms of $OA$ and $OP$.

(b) In the figure, Line$(P, Q)$ is tangent to $Circle(O_1, O_1P)$ and $Circle(O_2, O_2Q)$. Express $PQ$ in terms of $O_1P$ and $O_2Q$ and $O_1O_2$.

(c) In the figure, Line$(S, T)$ is tangent to $Circle(O_1, O_1S)$ and $Circle(O_2, O_2T)$. Express $ST$ in terms of $O_1S$ and $O_2T$ and $O_1O_2$.

Hints:
(i) Let $U$ be the point such that quad$(O_1STU)$ is a rectangle.
(ii) Use Pythagorean Theorem to relate the lengths of the sides of $\triangle O_1O_2U$.
(iii) Express the lengths of each side of $\triangle O_1O_2U$ in terms of the lengths $O_1S$ and $O_2T$ and $ST$.
(iv) Substitute lengths from (iii) into the equation from (ii).
(v) Solve for $ST$. 

Problems [7] and [8] are construction problems based on textbook Example 3.5.

In Example 3.5, given a segment of length $a$, a segment of length $a\sqrt{3}$ is constructed. Two different approaches are used: the “first approach” and the “second approach”. In the following two problems, we will use those same two approaches to construct a segment of length $a\sqrt{5}$.

[7] Given the segment of length $a$ shown below, construct a segment of length $a\sqrt{5}$ using the “first approach”. That is, using the equation $5 = 1^2 + 2^2$.

- Make your drawing big and clear.
- Label each object in your construction with just a number indicating when in the sequence the object was drawn.
- Make a separate list of the object numbers with a description of what each object is and how it was constructed.

[8] Given the segment of length $a$ shown above, construct a segment of length $a\sqrt{5}$ using the “second approach”. That is, using the equation $5 + 2^2 = 3^2$.

- Make your drawing big and clear.
- Label each object in your construction with just a number indicating when in the sequence the object was drawn.
- Make a separate list of the object numbers and a description of what each object is and how it was constructed.