Math 306 Homework Set #3, Due Friday, October 2, 2009

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[1] (Similar to suggested problem 3.1 # 20) Show how a direct proof of Statement A would begin and end. That is, show the proof structure. You do not have to fill in the proof.

Statement A: For all real numbers $x$, if $x > 1$ then $x^2 > x$.

[2] (Similar to suggested problems 3.1 # 31, 39, 47) Prove or disprove Statement B.

Statement B: There exists an integer $n$ such that $6n^2 + 27$ is prime.

[3] (Similar to suggested problems 3.1 # 31, 39, 47) Prove or disprove Statement C.

Statement C: For all integers $n$ and $m$, if $n - m$ is even then $n^3 - m^3$ is even.

Hint: To start, use long division to see if you can factor $(n - m)$ is a factor of $n^3 - m^3$.

[4] The Zero Product Property says that if a product of two real numbers is 0, then one of the numbers must be 0.

(a) Write this property formally using quantifiers and variables.

(b) Write the contrapositive of your answer to part (a).

(c) Write an informal version (without quantifier symbols or variables) for your answer to (b).

[5] (Similar to suggested problem 3.2 # 9) Suppose that $a$ and $b$ are both integers and that $a \neq 0$ and $b \neq 0$. Explain why $(b + a)/(a^2 b)$ must be a rational number. Hint: You will need to use [4c].

[6] Given two rational numbers $r$ and $s$ with $r < s$, prove that there is a rational number between $r$ and $s$. Hint: Use the results of 3.2 # 17, 18.

[7] (Similar to suggested problem 3.3 #15)

Prove that for all integers $a$, $b$, and $c$, if $a|b$ and $a|c$ then $a|(b - c)$.

[8] (Similar to suggested problems 3.3 #22, 24, 26) Prove or disprove Statement D.

Statement D: For all integers $a$, $b$, and $c$, if $a|bc$ then $a|b$ or $a|c$.

[9] How many zeros are at the end of $45^6 \cdot 44^{13}$? Explain how you can answer this question without actually computing the number (Hint: 10 = 2·5.)

[10] (Similar to suggested problem 3.3 #44) Prove that for any nonnegative integer $n$, if the sum of the digits of $n$ is divisible by 3, then $n$ is divisible by 3. (Hint: Solve 44, use it as a model.)