Math 306 (Barsamian) Class Drill 2.3.2
This Class Drill explores some of the concepts that are explored in textbook exercises 2.3 # 3, 418, 19, 21, 22, and 40.

For each pair of statements, do the following:

i. Which of the statements are true? (might be none, one, or both) Explain

ii. One of the statements is a famous property of real numbers. Which statement, and what is the name of the property? Explain

iii. Find the negation of any of the statements that are false.

Pair #1: Statement A: \( \forall x \in \mathbb{R}, \exists y \in \mathbb{Z} \) such that \( x < y \)
Statement B: \( \exists y \in \mathbb{Z} \) such that \( \forall x \in \mathbb{R}, x < y \)

Pair #2: Statement A: \( \forall x \in \mathbb{R}, \exists y \in \mathbb{R} \) such that \( x + y = x \)
Statement B: \( \exists y \in \mathbb{R} \) such that \( \forall x \in \mathbb{R}, x + y = x \)
Pair #3  Statement A: \( \forall x \in \mathbb{R}, \exists y \in \mathbb{R} \) such that \( x + y = 0 \)
Statement B: \( \exists y \in \mathbb{R} \) such that \( \forall x \in \mathbb{R}, x + y = 0 \)

Pair #4  Statement A: \( \forall x \in \mathbb{R}^*, \exists y \in \mathbb{R}^* \) such that \( xy = x \)
Statement B: \( \exists y \in \mathbb{R}^* \) such that \( \forall x \in \mathbb{R}^*, xy = x \)

Pair #5  Statement A: \( \forall x \in \mathbb{R}^*, \exists y \in \mathbb{R}^* \) such that \( xy = 1 \)
Statement B: \( \exists y \in \mathbb{R}^* \) such that \( \forall x \in \mathbb{R}^*, xy = 1 \)