Math 163A Handout 4: Definitions of Rates of Change

Definition of average rate of change
- words: the average rate of change of $f$ as the input changes from $a$ to $b$
- usage: $f$ is a function that is continuous on the interval $[a,b]$.
- meaning: the number $m = \frac{f(b) - f(a)}{b-a}$
- graphical interpretation: $m$ is the slope of the secant line containing points $(a, f(a))$ and $(b, f(b))$.
- remark: The average rate of change $m$ is a number.

Definition of instantaneous rate of change
- words: the instantaneous rate of change of $f$ at $a$.
- alternate words: the derivative of $f$ at $a$.
- symbol: $f'(a)$
- meaning: the number $m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$
- graphical interpretation: $m$ is the slope of the line tangent to the graph of $f$ at the point $(a, f(a))$.
- remark: The instantaneous rate of change $f'(a)$ is a number.

Definition of the Derivative
- words: the derivative of $f$
- symbol: $f'$
- meaning: Note that $f'$ is a function. To describe the function, I have to tell you how it produces the output for a given input. For an input $x$, the output $f'(x)$ is the number $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.
- graphical interpretation: For an input $x$, the output $f'(x)$ is the number that is the slope of the line tangent to the graph of $f$ at the point $(a, f(a))$.
- remark: The derivative $f'$ is a function.

Physics Terminology of Position and Velocity
- Time: When using mathematical functions to describe the motion of objects, $x$ is a variable that represents the elapsed time. (In physics books, the letter $t$ is used, but our book uses $x$.)
- Position: In physics, to say an object is “moving in 1 dimension” means that it can go forwards or backwards but can’t turn. Examples are a train moving on a straight track, a car moving on a straight road, or an object going straight up & down under the influence of gravity. In such situations, a single coordinate can be used to keep track of the position of the object. A function called the position function gives the value of the coordinate at a given time. In our book, the position function is called $f$. That is, at time $x$, the coordinate of the position of the object is the number $f(x)$. (In physics books, the letter $s$ is used for the position function. So at time $t$, the coordinate of the position of an object is $s(t)$.)
- average velocity: The words “the average velocity from time $x = a$ to time $x=b$” mean the same thing as the words “the average rate of change of $f$ as the input changes from $a$ to $b$”.
- instantaneous velocity: The words “the instantaneous velocity at time $x =a$” mean the same thing as the words “the instantaneous rate of change of $f$ at time $x = a$.” That is, the number $f'(a)$. (In physics, the letter $v$ is used for the velocity, so the symbol would be $v(a)$.)
- velocity: The word “velocity” means the same thing as the words “the derivative of the position function”. That is, the velocity is the function $f'$. (In physics, the symbol would be $v$.)