New Rule of Inference: The Principle of Induction

\[ P(a) \text{ is true} \]
\[ \text{For all integers } k \geq a, \text{ if } P(k) \text{ is true, then } P(k+1) \text{ is true.} \]
\[ \therefore \text{For all integers } n \geq a, \ P(n) \text{ is true.} \]

Usage:
- The letter “a” represents some fixed integer.
- The letters “k” and “n” represent variables whose domain \( D \) is the set of all integers greater than or equal to “a”.
- The symbol \( P(n) \) represents a predicate whose domain is the set \( D \).

This new rule of inference will be used to prove statements of the form
“For all integers \( n \geq a \), \( P(n) \) is true.”

Strategy for using the principle of induction

Preliminary work:
- Identify the number playing the role of “a”. (Introduce it.)
- Identify the predicate \( P(n) \). (Introduce it in a sentence.)
- Figure out what the expressions for \( P(a) \), \( P(k) \), and \( P(k+1) \) look like. (Write them down.)

Build a proof using the following structure:

Proof that for all integers \( n \geq a \), \( P(n) \) is true:

Basis Step: Prove that \( P(a) \) is true.

* * a bunch of steps may be involved
* * Inductive Step: Prove that for all integers \( k \geq a \), if \( P(k) \) is true, then \( P(k+1) \) is true.

* * a bunch of steps may be involved
* * Conclusion: Therefore, for all integers \( n \geq a \), \( P(n) \) is true. (by the principle of induction)

End of Proof