Handout 08: Graphing a Polynomial with the 10-Step Method
Math 163A Fall 2006 Section 02 (Barsamian)

Use the 10-step method to produce a graph of the function \( f(x) = x^3 - 9x^2 + 15x - 7 \).

\[
f(x) = x^3 - 9x^2 + 15x - 7 = \left[ (x-1)^2 \right] (x-7)
\]

You may use the following information:

\[
f'(x) = 3x^2 - 18x + 15 = 3(x-1)(x-5)
\]

\[
f''(x) = 6x - 18 = 6(x-3)
\]

Solution

Step 1: \( f(0) = (0)^3 - 9(0)^2 + 15(0) - 7 = -7 \), so the point \( (0, -7) \) is the \( y \)-intercept.

Step 2: To check for symmetries, we compare the three functions \( f(x) \), \( f(-x) \), and \( -f(-x) \).

\[
\begin{align*}
f(x) &= x^3 - 9x^2 + 15x - 7 \\
f(-x) &= (-x)^3 - 9(-x)^2 + 15(-x) - 7 = -x^3 - 9x^2 - 15x - 7 \\
-f(-x) &= -(x^3 - 9x^2 - 15x - 7) = x^3 + 9x^2 + 15x + 7
\end{align*}
\]

Because none of these match we conclude that the graph of \( f \) will not have \( y \)-axis or origin symmetry.

Step 3: The end behavior will resemble \( y = x^3 \). That is, “down on the left, up on the right.

Step 4: \( f(x) = x^3 - 9x^2 + 15x - 7 = \left[ (x-1)^2 \right] (x-7) \). The sign chart for \( f \) is:

\[
\begin{array}{cccccc}
& f \text{ negative} & f \text{ zero} & f \text{ negative} & f \text{ zero} & f \text{ positive} \\
& [+](-) & [0](-) & [+](-) & [+](0) & [+](+)
\end{array}
\]

sign chart for \( f(x) = x^3 - 9x^2 + 15x - 7 = \left[ (x-1)^2 \right] (x-7) \)

Step 5:

- The linear factor \( (x-1) \) appears in the numerator and not the denominator. Therefore, step 5 case 1 applies: the graph of \( f \) will have an \( x \)-intercept at \( x = 1 \). (This is confirmed by the sign chart. Good.)
- The linear factor \( (x-7) \) appears in the numerator and not the denominator. Therefore, step 5 case 1 applies: the graph of \( f \) will have an \( x \)-intercept at \( x = 7 \). (This is confirmed by the sign chart. Good.)

Step 6 & 7: \( f''(x) = 3x^2 - 18x + 15 = 3(x-1)(x-5) \). The sign chart for \( f'' \) is:

\[
\begin{array}{cccccc}
& f \text{ positive} & f \text{ increasing} & f \text{ negative} & f \text{ decreasing} & f \text{ positive} \\
& +(-)(-) & +(0)(-) & +(+)(-) & +(+)0 & +(+) (+)
\end{array}
\]

sign chart for \( f''(x) = 3x^2 - 18x + 15 = 3(x-1)(x-5) \)
We see that the numbers $x = 1$ and $x = 5$ are critical numbers for $f$. We already know that the point $(1,0)$ is on the graph. But we still need to find the $y$-value corresponding to $x = 5$. We find that $f(5) = -32$. Therefore, there will be a critical point at $(5,-32)$.

**Step 8 & 9:** $f''(x) = 6x - 18 = 6(x - 3)$. The sign chart for $f''$ is:

```
+(-)  (+0)  +(+)  
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>f'' negative</td>
<td>f'' zero</td>
<td>f'' positive</td>
</tr>
<tr>
<td>f concave down</td>
<td>f not concave</td>
<td>f concave up</td>
</tr>
</tbody>
</table>
```

The $y$-value at the inflection point is $f(3) = -16$. So the inflection point is at $(3,-16)$.

**Step 10:** Here is the graph of $f(x) = x^3 - 9x^2 + 15x - 7$. The points of interest are:

- $(1,0)$
- $(0,-7)$
- $(3,-16)$
- $(5,-32)$
- $(7,0)$