Calculators are not allowed on this exam.

(20 points) (a) Find all values of $t$ in the interval $[0, 2\pi]$ that satisfy the equation $\sin(t) = -\frac{1}{2}$. (Draw a picture that illustrates how you got your answer.)

(b) Find all values of $t$ in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ that satisfy the equation $\sin(t) = -\frac{1}{2}$. (Draw a picture that illustrates how you got your answer.)

(c) $\arcsin\left(-\frac{1}{2}\right) =$
(20 points) (a) Find a cosine function whose graph would look like the one shown to the right. (The actual graph extends forever to in the positive and negative $t$ directions.) Explain your answer.

(b) Find a sine function whose graph would look like the one above. Explain your answer.
The goal of this problem is to produce a graph of \( t = \arctan(2r - 1) \) by using transformations of graphs.

(a) Graph \( r = \tan(t) \). (This is the “Capital” tangent function, the one with the restricted domain.) Label all important points and asymptotes.

(b) Graph \( t = \arctan(r) \). (Remember that the “arctan” function is the inverse function for “Tan”. So, you can obtain this graph by flipping the first graph in a certain way.) Label all important points and asymptotes.

(c) Graph \( t = \arctan(r - 1) \). Label all important points and asymptotes.

(d) Graph \( t = \arctan(2r - 1) \).
The goal of this problem is to produce a graph of $x = 2\log_5(y) - 2$ by using transformations of graphs.

(a) Graph $y = 5^x$. Label all important points and asymptotes.

(b) Graph $x = \log_5(y)$. (Remember that the “$\log_5(\ )$” function is the inverse function for “$5(\ )$”. So, you can obtain this graph by flipping the first graph in a certain way.) Label all important points and asymptotes.

(c) Graph $x = 2\log_5(y)$. Label all important points and asymptotes.

(d) Graph $x = 2\log_5(y) - 2$. Label all important points and asymptotes.
(30 points) The radioactive isotope thorium 234 has a half-life of approximately 578 hours.

(a) If a sample has an initial mass of 50 milligrams, find an expression for the mass after \( t \) hours.

(b) How much will remain after 100 hours?

(c) When will the initial mass decay to 10 milligrams?

(10 Points) Sketch an ellipse that has \( x \)-intercepts at \((5,0)\) and \((-5,0)\), and has \( y \)-intercepts at \((0,4)\) and \((0,-4)\). Find the coordinates of the foci for this ellipse, and put them on the graph, as well.
(40 points) The goal for this problem is to give an equation for a hyperbola that foci at \((0, 5)\) and \((0, -5)\), and vertices at \((0, 4)\) and \((0, -4)\), and then to sketch it.

(a) For this problem, \(c = 5\) and \(a = 4\). What is \(b\)? Explain.

(b) What is the equation that describes the hyperbola? Explain.

(c) What are the equations that describe the asymptotes? Explain.

(d) Sketch the hyperbola, showing the important points and the asymptotes.