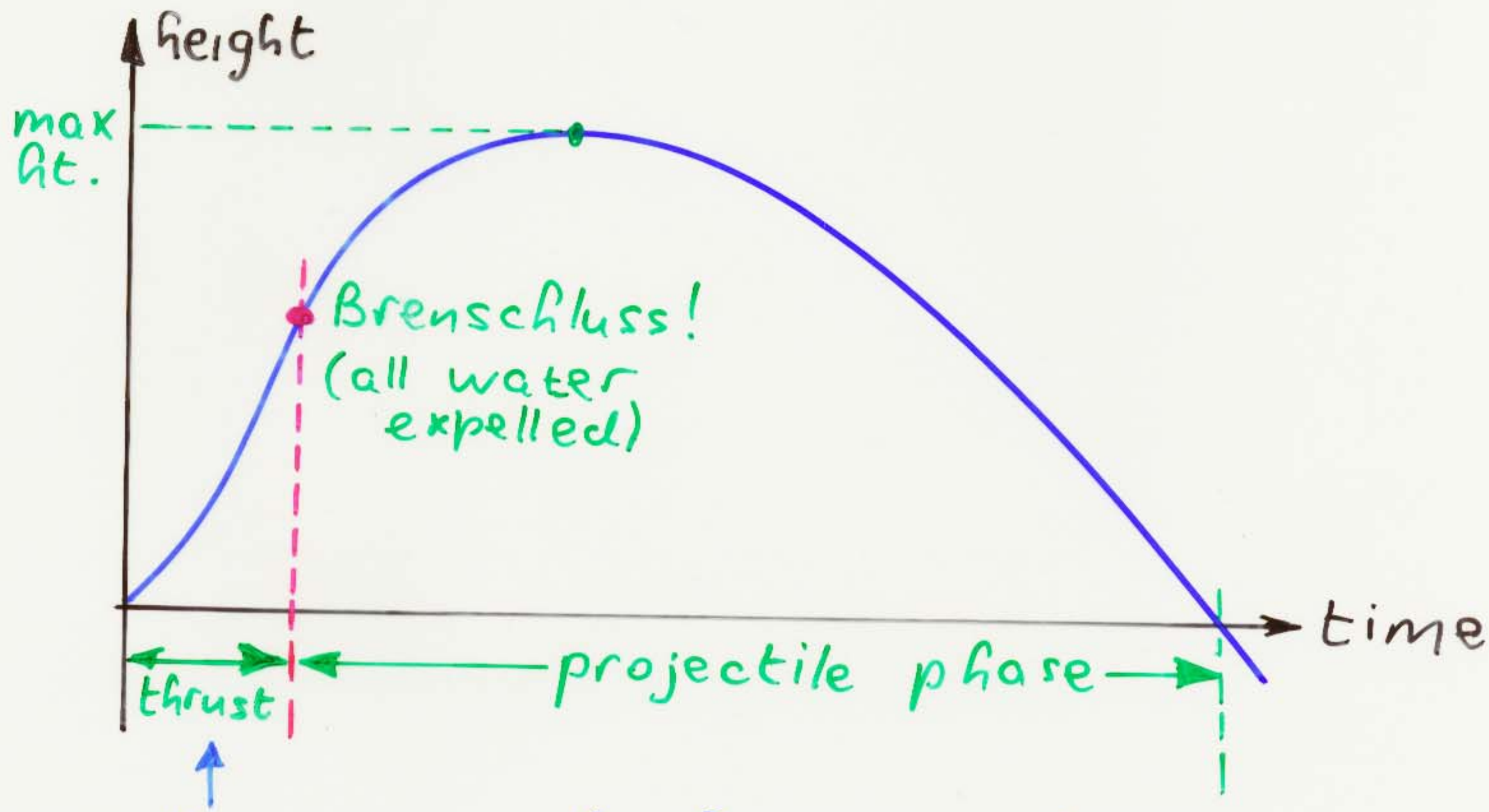
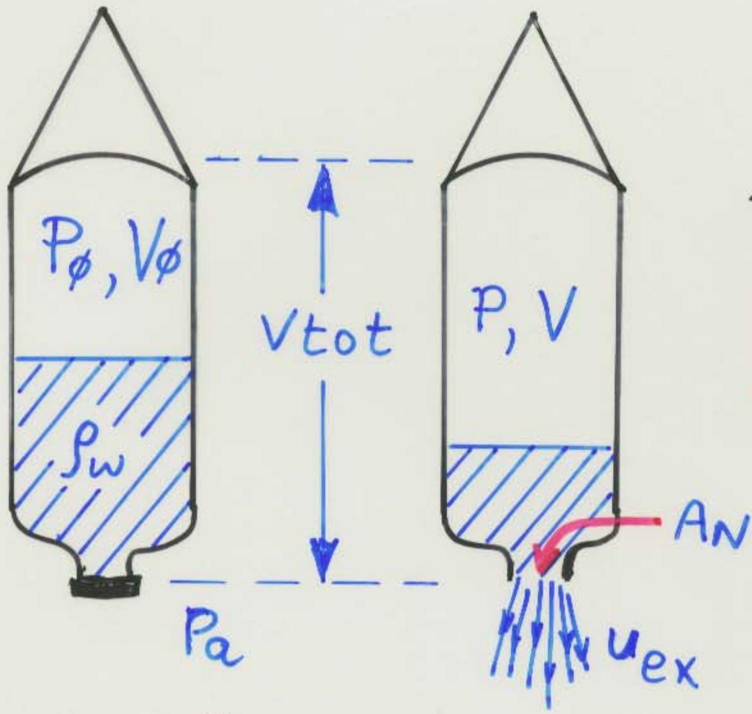
Two Phases of Rocket Flight

1 - **Thrust Phase** - rocket accelerates upwards while water propellant is expelled under pressure.

2 - **Projectile Phase** - rocket slows down under air drag and gravity, reaches maximum height, and finally returns to earth.



Determine the Duration of Thrust



Thrust Phase

$$\boxed{V < V_{tot}}$$

$\Downarrow$

determine

$V(t)$

1. Bernoulli :

Flow work done to expell the water

= Increase in kinetic energy of the water

$$\boxed{\frac{P - P_a}{\rho_w} = \frac{u_{ex}^2}{2}} \qquad (1)$$

2. Rate of increase of air volume

= volumetric flow rate of water through nozzle

$$\boxed{\frac{dV}{dt} = A_N . u_{ex}} \qquad (2)$$

Thus combining (1) and (2)

$$\boxed{\frac{dV}{dt} = A_N \sqrt{\frac{2(P - P_a)}{\rho_w}}}$$

<u>Note:</u>
P is a Variable

Assuming adiabatic expansion
(no heat flow during the process)

$$\boxed{P = P_\emptyset \left(\frac{V_\emptyset}{V} \right)^k}$$

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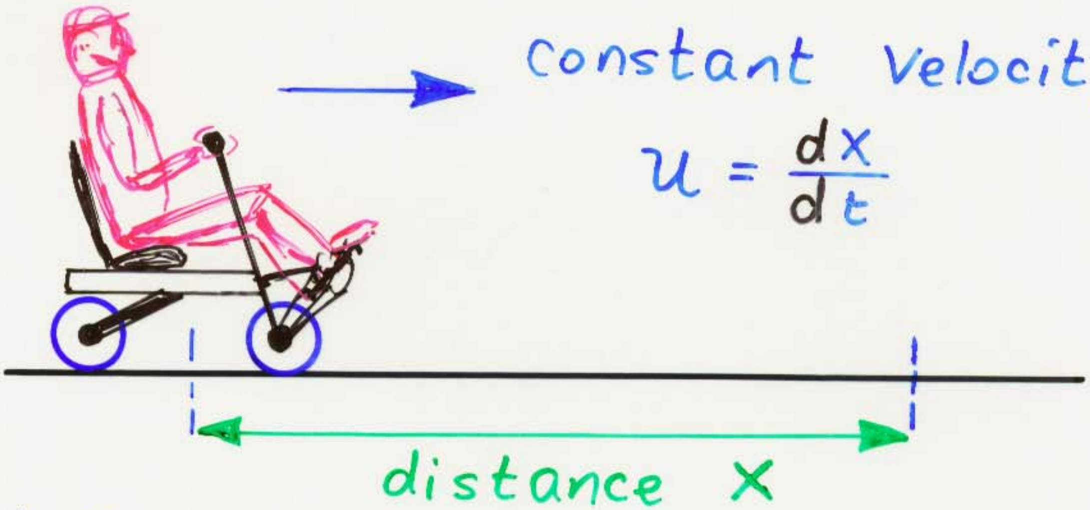
Thus

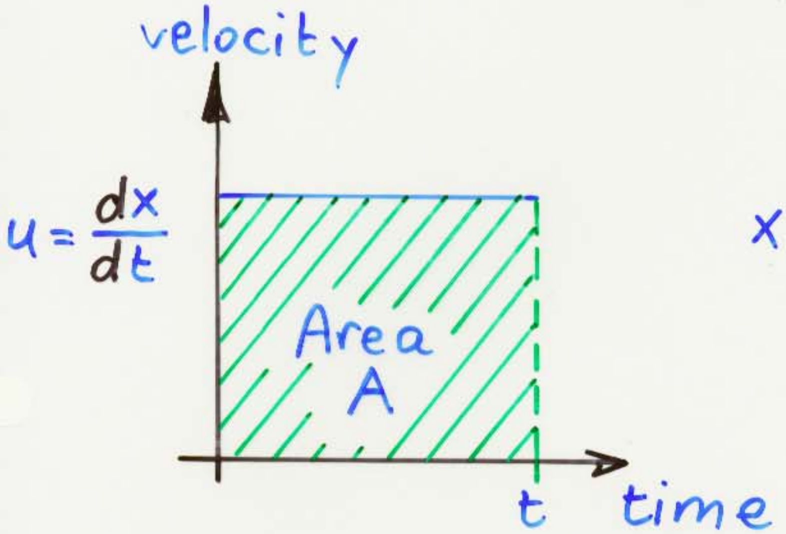
$$\boxed{\frac{dV}{dt} = A_N . \sqrt{\frac{2\left[P_o \left(\frac{V_o}{V}\right)^k - P_a\right]}{\rho_w}}}$$

We need to <u>integrate</u> to determine
V(t), however V is deeply
imbedded in the differential
equation in a nonlinear manner

constant velocity

$$u = \frac{dx}{dt}$$



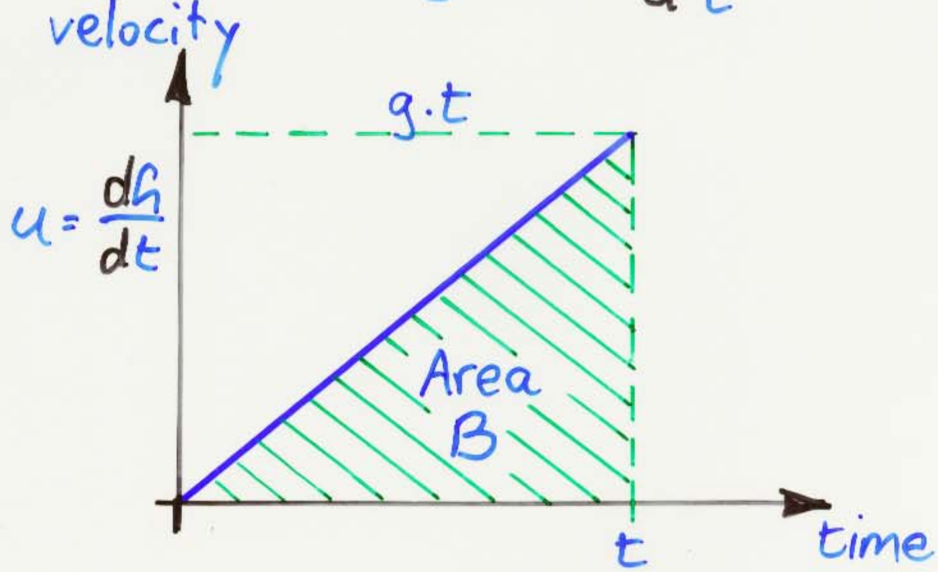


$$x = \int_0^t u.dt = u.t$$

$\underline{x = u.t = \text{Area } A}$ under the curve

Acceleration $g = 9.81 \frac{m}{s^2}$

$$u = g.t = \frac{dh}{dt}$$



$$h = \int_0^t u.dt$$

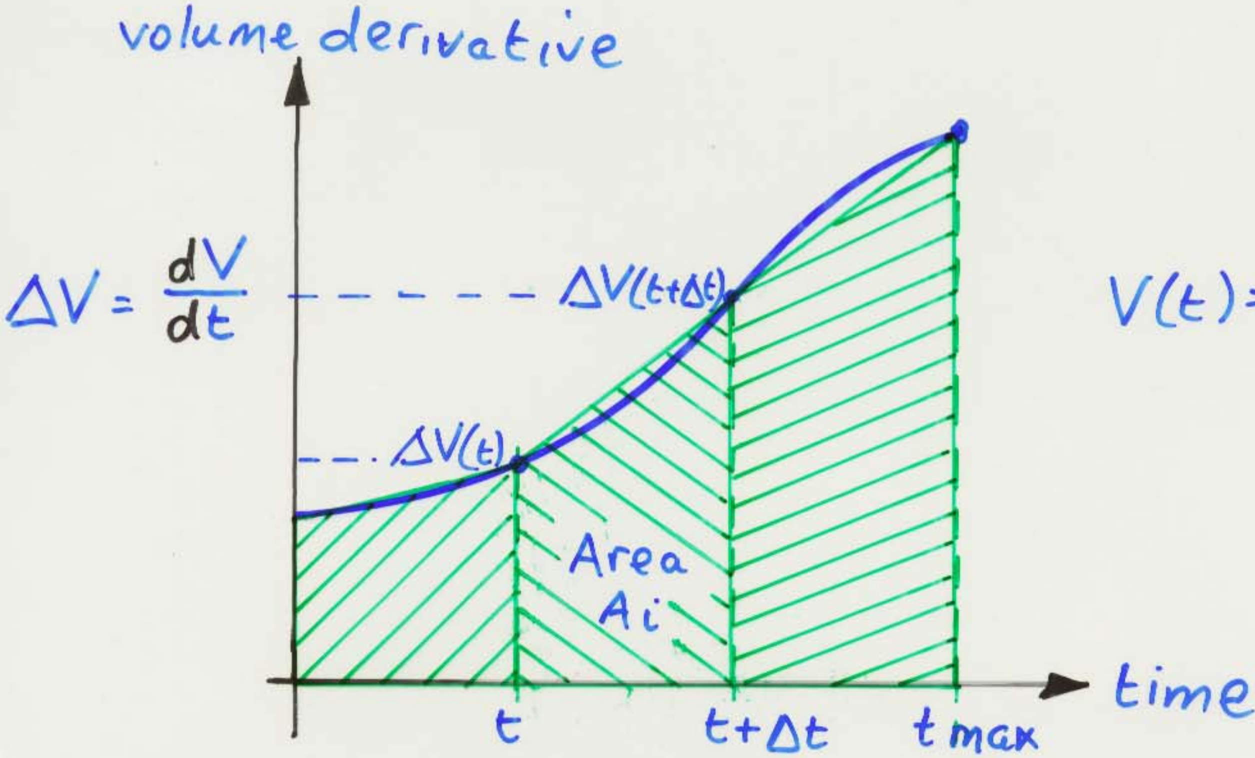
$$= \int_0^t gt.dt$$

$$= \frac{1}{2} g t^2$$

$h = \frac{1}{2} g t^2$ = Area B under the curve

Numerical Integration (Lab 5)

(Applying this to volume V)



$$V(t) = \sum_i Area_i$$

Trapezoidal Rule

$$V(t+\Delta t) = V(t) + \left[\frac{\Delta V(t) + \Delta V(t+\Delta t)}{2}\right] \Delta t$$

where:

$$\Delta V(t) = \left.\frac{dV}{dt}\right|_t = A_N \cdot \sqrt{\frac{2\left[P_0\left(\frac{V_0}{V(t)}\right)^k - P_a\right]}{\rho_w}}$$

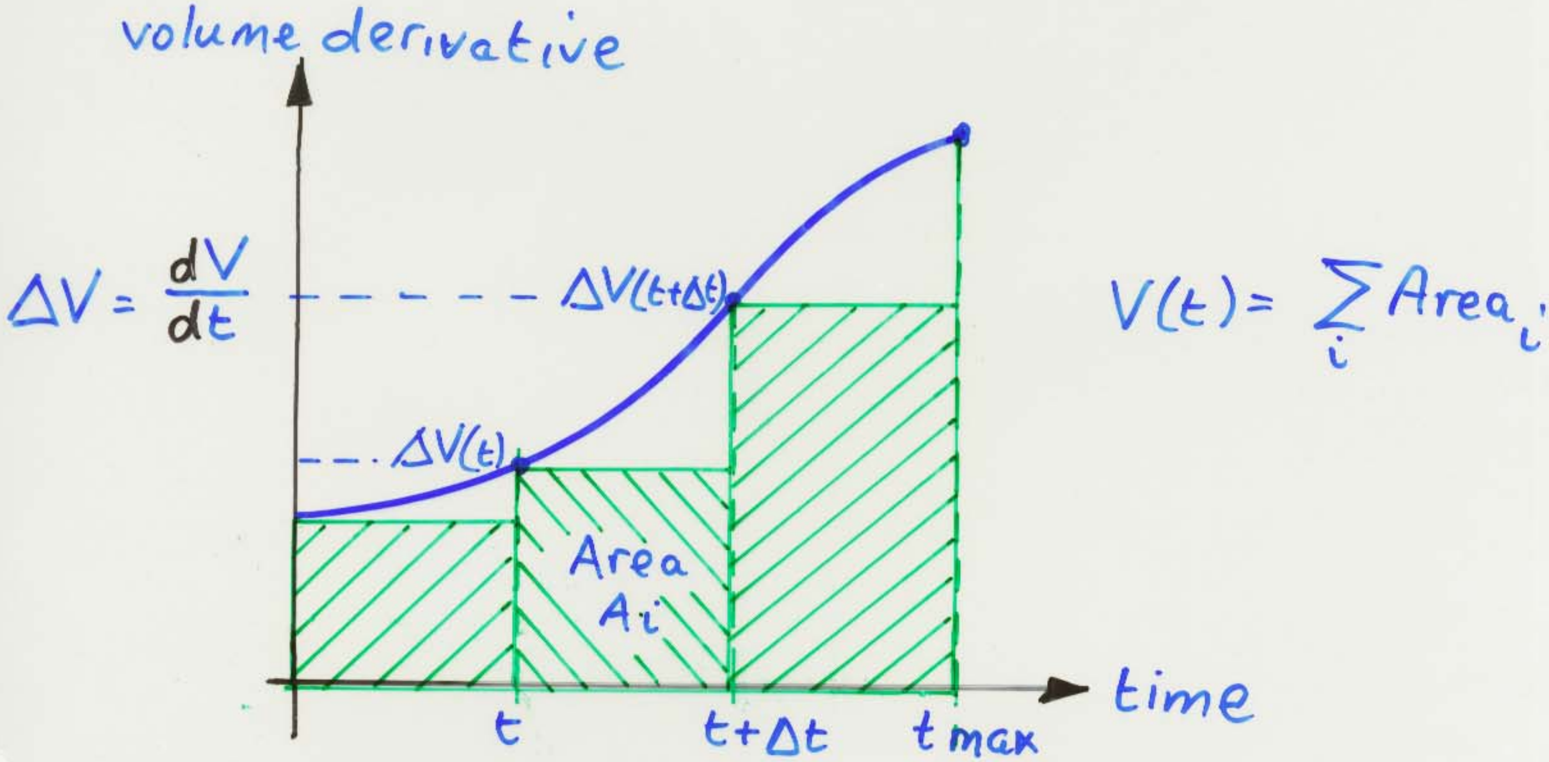
however:

We don't know $V(t+\Delta t)$

thus: we cannot use this method

Numerical Integration (Lab 5)

(Applying this to volume V)



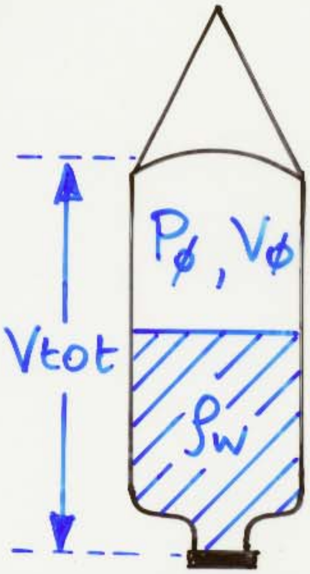
Euler's Method

$$V(t+\Delta t) = V(t) + \Delta V(t) . \Delta t$$

Assumption:

ΔV does not change significantly from t to $t+\Delta t$

findtime(tmax)



Initialize:

$P = P_\phi$

$V = V_\phi$

$t = \phi$

$$\Delta t = \frac{t_{max}}{1000} \; ?$$

Lab 5

$V < V_{tot}$?

No → return t

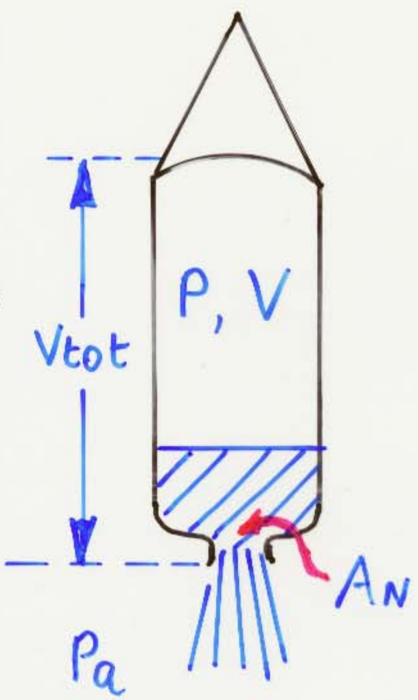
Yes

$t = t + \Delta t$

$$P = P_\phi \left(\frac{V_\phi}{V}\right)^k$$

$$\Delta V = A_N \cdot \sqrt{\frac{2(P - P_a)}{\rho_w}}$$

$V = V + \Delta V * \Delta t$



return t