OHIO
UNIVERSITY

# MATH 263C CCE — SAMPLE EXAMINATION

You will gain the most benefit from this sample exam if you take it as if it were the supervised examination. Set yourself a time limit of 3 hours and do not use your textbook or notes. Practice writing out all the steps in your solutions to the problems, since you will have to show your work on the problems for the supervised examination.

1. Determine if the following sequences $\{a_n\}$ converge or diverge, and if a sequence converges, find $\lim_{n\to\infty} a_n$.

   (1) $a_n = \dfrac{n}{n+1}\left(1 - \dfrac{1}{n^2}\right)$;

   (2) $a_n = (-1)^n \dfrac{n}{n+1}$;

   (3) $a_n = n \sin \dfrac{1}{n}$

2. Determine which of the following series are convergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n + \cos n}{n^3 + 1}$;

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + 1}$.

3. Determine if the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$$

   is absolutely convergent, conditionally convergent, or divergent.

4. (a) Prove that the series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{(2n+1)!}$$

   converges absolutely.

(b) The series

$$\sum_{n=1}^{\infty}\left(\frac{2x}{3}\right)^{n}$$

is a geometric series. Find the interval of convergence and the sum of the series.

5. Find the interval of convergence of the series

$$\sum_{n=1}^{\infty}\frac{1}{n(2^{n})}(x+10)^{n}$$

6. Use the fact that $e^{x}$ has the power series representation

$$e^{x}=1+x+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}+\ldots+\frac{x^{n}}{n!}+\ldots$$

to find a power series representation of:

(a) $\sinh x=\dfrac{e^{x}-e^{-x}}{2}$

(b) $x^{2}e^{(x^{2})}$

7. Use the first two terms of an appropriate infinite series and approximate the integral $\int_{0}^{0.5}e^{-(x^{3})}dx$.

***Hint:*** Note the power series in powers of $x$ for $e^{x}$ is

$$e^{x}=1+x+\frac{x^{2}}{2!}+\ldots+\frac{x^{n}}{n!}+\ldots=\sum_{n=0}^{\infty}\frac{x^{n}}{n!}.$$

8. (a) Write the first three non-zero terms in the Taylor series for $f(x)=\sin x$ at $x=\pi/2$.

(b) Write the first three terms in the Maclaurin series for $f(x)=(1+x)^{1/5}$.

9. (a) Find the rectangular coordinate equation of the conic having parametric equations $x=2\sin t,\ y=3\cos t,$ where the parameter $t$ satisfies $0\le t\le 2\pi$.

(b) Find the equation of the parabola whose vertex is at the origin (0,0), and whose axis is the $y$-axis, if the parabola passes through the point $(-3,5)$. Then determine the coordinates of the focus of the parabola, and the equation of the directrix.

10. For the hyperbola with equation

$$\frac{x^{2}}{16}-\frac{y^{2}}{9}=1,$$

(a) Find the coordinates of the foci, and the equation of the asymptotes.

(b) Sketch a rough graph of the hyperbola, including the asymptotes.

11. (a) Sketch a rough graph of the polar coordinate equation $r = 2 - \sin\theta$.

(b) The graph of $r = -2\sin\theta$ (polar coordinates) is a circle. Sketch the graph of the circle, and find its rectangular coordinate equation.

12. (a) Find the area of the region bounded by the graph of $r^2 = 2\sin 2\theta$ (a lemniscate; that is, like a "figure eight").

(b) *Set up* (do not evaluate) an integral (use polar coordinates) that gives the area of the region in the *x*, *y* plane that is:

(i) inside the graphs of both the circles $r = 2$ and $r = 4\cos\theta$.

(ii) outside the graph of $r = 2$, and inside the graph of $r = 4\cos\theta$.

13. (a) Describe briefly the graph in 3-space of each of the following equations:

(1) $y = 3$;

(2) $x = y$;

(3) $x^2 + y^2 = 4$;

(4) $z = \sqrt{9 - x^2 - y^2}$;

(5) $xy = 0$.

(b) For the points $P_1\,(0,0,1)$ and $P_2\,(1,3,4)$ in a rectangular $x, y, z$ coordinate system, find:

(i) a vector $\vec{a} = \langle a_1, a_2, a_3\rangle$ whose position vector represents the vector $\overrightarrow{P_1 P_2}$.

(ii) the symmetric equations of a line through the points $P_1$ and $P_2$.

14. For vectors $\vec{a} = \langle 1, 1, 0\rangle$ and $\vec{b} = \langle 0, 1, 1\rangle$, find the angle $\theta$ between $\vec{a}$ and $\vec{b}$.

15. Find a unit tangent vector to the curve $x = 2 + \sin t,\ y = \cos t,\ z = t$ (parameter *t*) at the point $P\,(2, 1, 0)$.

16. (a) Compute the cross product $\vec{a}\ x\ \vec{b}$ of the vectors

$$\vec{a} = 4\vec{i} + 3\vec{j} - \vec{k}, \quad \vec{b} = 2\vec{i} - 5\vec{j} + 6\vec{k}.$$

(b) Using your result for $\vec{a}\ x\ \vec{b}$ from part 16(a), find the equation of the plane through $(2, -3, 2)$ and parallel to the plane containing the vectors $\vec{a}$ and $\vec{b}$.

# ANSWER KEY TO SAMPLE EXAMINATION

1. (a) $a_n = \frac{n}{n+1}\left(1 - \frac{1}{n^2}\right) \Rightarrow \lim_{n\to\infty} a_n = \left(\lim_{n\to\infty} \frac{n}{n+1}\right) \cdot \lim_{n\to\infty}\left(1 - \frac{1}{n^2}\right) = 1 \cdot 1 = 1.$

In $a_n = (-1)^n \frac{n}{n+1}$, note the terms are:

$$-\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, -\frac{5}{6}, \frac{6}{7}, \ldots$$

Clearly, the terms oscillate back and forth between positive and negative values; so the sequence diverges. Alternatively, note there are two sub-sequences which do converge. The sub-sequence $-\frac{1}{2}, -\frac{3}{4}, -\frac{5}{6}, \ldots$ converges to $-1$, and the sub-sequence $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \ldots$ converges to 1. Since there are two infinite subsequences converging to *different* values, the given sequence diverges.

To handle $a_n = n \sin\frac{1}{n}$, where $n$ is a positive integer, define $f(x) = \frac{\sin\frac{1}{x}}{\frac{1}{x}}$, for $x > 0$.

***Note:*** letting $z = \frac{1}{x}$, that $\lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = \lim_{z\to 0^+} \frac{\sin z}{z} = 1$. Therefore, $\lim_{n\to\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}} = 1$.

Alternatively, you can use L'Hôpital's Rule. Thus,

$$\lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\left(-\frac{1}{x^2}\right)\cos\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to\infty} \cos\frac{1}{x} = \cos 0 = 1.$$

Hence, $\lim_{n\to\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} n \sin\frac{1}{n} = 1.$

2. (a) First, observe that the series

$$\sum_{n=1}^{\infty} \frac{n+\cos n}{n^3+1}$$

is a positive-term series. It is easy to see, intuitively, that this series converges, because for large $n$, $(n + \cos n)/(n^3 + 1) \approx n/n^3 = 1/n^2$, and $\sum_{1}^{\infty} 1/n^2$ converges ($p$-series, $p = 2 > 1$). To be more rigorous, which is the idea here, you can prove the given series converges using either the ordinary comparison test, or the limit comparison test.

Using the Ordinary Comparison test, observe that

$$\frac{n + \cos n}{n^3 + 1} \le \frac{n + 1}{n^3} \le \frac{2n}{n^3} = \frac{2}{n^2}$$

for every positive integer $n$. Since $\sum \frac{2}{n^2}$ converges (because $\sum 1/n^2$ does, as it is a $p$-series with $p = 2 > 1$), then the given series converges, by the Ordinary Comparison test.

To apply the Limit Comparison test in testing the given series $\sum \frac{n + \cos n}{n^3 + 1} = \sum a_n$, note that for large $n$, the $n$-th term $a_n$ is essentially like $b_n = n/n^3 = 1/n^2$. Further,

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n + \cos n}{n^3 + 1}(n^2)$$

$$= \lim_{n\to\infty} \frac{n^3 + n^2 \cos n}{n^3 + 1} = \lim_{n\to\infty}\left[\frac{\left(1 + \frac{\cos n}{n}\right)}{1 + 1/n^3}\right] = 0$$

Therefore, since $\sum b_n = \sum 1/n^2$ converges, so does the given series $\sum a_n$, by the Limit Comparison test.

2. (b) The given series

$$\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + 1}$$

diverges, because $\lim_{n\to\infty} \frac{n^2 - 1}{n^2 + 1} = 1 \ne 0$.

3. To test $\sum_1^\infty (-1)^n \frac{n}{n^2+1}$ for absolute convergence, consider $\sum_1^\infty \frac{n}{n^2+1}$. Since $\frac{n}{n^2+1} \geq \frac{n}{n^2+n^2} = \frac{1}{2n}$, for positive integers $n \geq 1$, and $\sum 1/2\,n$ diverges (a harmonic series, except for a constant factor), then the given series is not absolutely convergent. However, the given series is a convergent alternating series, since $\lim_{n\to\infty} \frac{(-1)\,n}{n^2+1} = 0$, and the sequence $\left\{\frac{n}{n^2+1}\right\} = \left\{a_n\right\}$, with $a_1 = \frac{1}{2}$, $a_2 = \frac{2}{5}$, $a_3 = \frac{3}{10}$, … is decreasing. Thus, the given series is conditionally convergent.

4. (a) Apply the Absolute Ratio test to the given series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{(2n+1)!} = \sum_{n=1}^{\infty} u_n.$$

We have $p = \lim_{n\to\infty} \left|\frac{u_n + 1}{u_n}\right| = \lim_{n\to\infty} \frac{1}{[2\,(n+1)+1]!} \cdot (2n+1)!$

$$= \lim_{n\to\infty} \left[\frac{(2n+1)!}{(2n+3)!}\right. = \lim_{n\to\infty} \frac{1}{(2n+3)(2n+2)} = 0 < 1$$

Therefore, the given series converges absolutely, and hence converges.

(b) Recall that a geometric series with ratio $r$, namely

$$a + ar + ar^2 + \ldots = \sum_{k=1}^{\infty} a_r^{k-1}$$

converges and has sum $a/(1-r)$ if $|r| < 1$. Now the given series

$$\sum_{n=1}^{\infty} \left(\frac{2x}{3}\right)^n = \frac{2x}{3} + \left(\frac{2x}{3}\right)^2 + \left(\frac{2x}{3}\right)^3 + \ldots$$

is a geometric series with ratio $2x/3$. This series converges if $\left|\frac{2x}{3}\right| < 1 \Leftrightarrow |x| < 3/2 \Leftrightarrow -3/2 < x < 3/2$; for these values of $x$, that is, for values of $x$ in the interval of convergence $(-3/2,\ 3/2)$, the sum of the series is $\frac{2x/3}{1 - \frac{2x}{3}} = \frac{2x}{3-2x}$.

5. Applying the Absolute Ratio test to

$$\sum_{n=1}^{\infty} \frac{(x+10)^n}{n(2^n)}, \text{ we get}$$

$$p = \lim_{n\to\infty} \left| \frac{(x+10)^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n(2^n)}{(x+10)^n} \right| = \lim_{n\to\infty} \left| (x+10) \frac{n}{(n+1)(2)} \right|$$

$$= |x+10|/2$$

Hence, the given series converges absolutely (and therefore converges) for $\frac{|x+10|}{2} < 1 \Leftrightarrow |x+10| < 2 \Leftrightarrow -2 < (x+10) < 2 \Leftrightarrow -12 < x < -8.$

Checking the endpoints $x = -12$, the given series is

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n(2^n)} = \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n(2^n)} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n},$$

which converges by the Alternating Series test. For $x = -8$, the given series is

$$\sum_{n=1}^{\infty} \frac{2^n}{n(2^n)} = \sum_{n=1}^{\infty} \frac{1}{n},$$

which diverges (harmonic series). Therefore, the interval of convergence of the given series is the interval $[-12, -8)$.

6. (a) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$

Hence,

$e^x - e^{-x} = 2x + \frac{2x^3}{3!} + \frac{2x^5}{5!} + \ldots$, and

$\sinh x = (e^x - e^{-x})/2 = x + x^3/3! + x^5/5! + \ldots$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}, \quad -\infty < x < \infty$$

(b) $x^2 e^{x^2} = x^2 \left( 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \ldots \right) = x^2 + x^4 + \frac{x^6}{2!} + \frac{x^8}{3!} + \ldots$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+2}}{n!}, \quad -\infty < x < \infty.$$

7. First, in the power series

$$e^{x} = 1 + x + \frac{x^{2}}{2!} + \ldots + \frac{x^{n}}{n!} + \ldots, \quad -\infty < x < \infty,$$

replace $x$ by $-x^3$. This gives

$$e^{-x^{3}} = 1 - x^{3} + \frac{x^{6}}{2!} - \frac{x^{9}}{3!} + \ldots,$$

and $\int_0^{0.5} e^{-x^3}\,dx = \int_0^{0.5} (1 - x^3 + \frac{x^6}{2!} - \frac{x^9}{3!} + \ldots)\,dx$

$$= x - \frac{x^{4}}{4} + \frac{1}{2}\cdot\frac{x^{7}}{7} - \frac{1}{3!}\,\frac{x^{10}}{10} + \ldots\Big]_0^{0.5}$$

$$\approx 0.5 - \frac{(0.5)^4}{4} = .5 - \frac{.0625}{4} \approx .484$$

with error from the true sum $< \frac{1}{2}\,\frac{(0.5)^7}{7} \approx .00056.$

8. Here, $f(x) = \sin x \Rightarrow f'(x) = \cos x,\ f''(x) = -\sin x,\ f'''(x) = -\cos x,\ f^{(4)}(x) = \sin x,\ \ldots.$

Now, $f(\pi/2) = \sin \pi/2 = 1;\ f'(\pi/2) = 0;\ f''(\pi/2) = -1$
$f'''(\pi/2) = 0,\ f^{(4)}(\pi/2) = 1,$ etc.

The Taylor series for $f(x) = \sin x$ in powers of $(x - \pi/2)$ is

$$f(x) = \sin x = \sum_{n=0}^{\infty} \frac{f^{(n)}(\pi/2)}{n!}(x - \pi/2)^n$$

$$= f\left(\frac{\pi}{2}\right) + f'\left(\frac{\pi}{2}\right)(x - \pi/2) + \frac{f''(\pi/2)}{2!}(x - \pi/2)^2$$

$$+ \frac{f'''(\pi/2)}{3!}(x - \pi/2)^3 + \frac{f^{(4)}(\pi/2)}{4!}(x - \pi/2)^4 + \ldots$$

$$= 1 - \frac{(x - \pi/2)^2}{2!} + \frac{(x - \pi/2)^4}{4!} + \ldots$$

(b) $f(x) = (1+x)^{1/5} = f(0) + f'(0)\,x + \dfrac{f''(0)}{2!}x^2 + \ldots$

Here, $f'(x) = \dfrac{1}{5}(1+x)^{-4/5}$, $f''(x) = -\dfrac{4}{25}(1+x)^{-9/5}$,

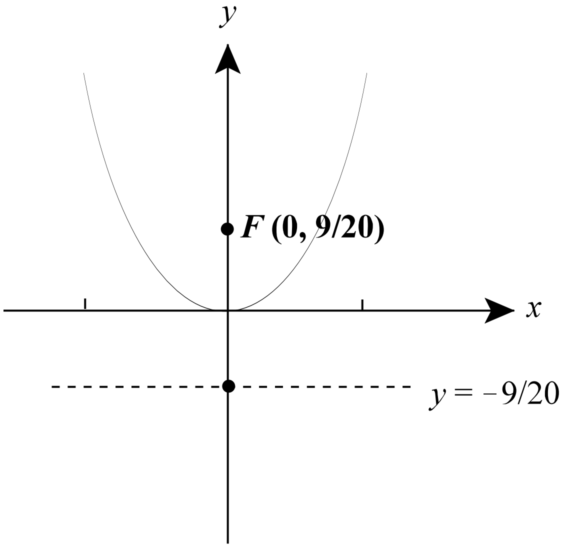
and $f(0) = 1$, $f'(0) = 1/5$, $f''(0) = -4/25$; so

$f(x) = 1 + \dfrac{1}{5}x - \dfrac{4}{50}x^2 + \ldots$

9. (a) $x = 2\sin t,\ y = 3\cos t \Leftrightarrow \left(\dfrac{x}{2}\right)^2 + \left(\dfrac{y}{3}\right)^2 = \sin^2 t + \cos^2 t = 1.$

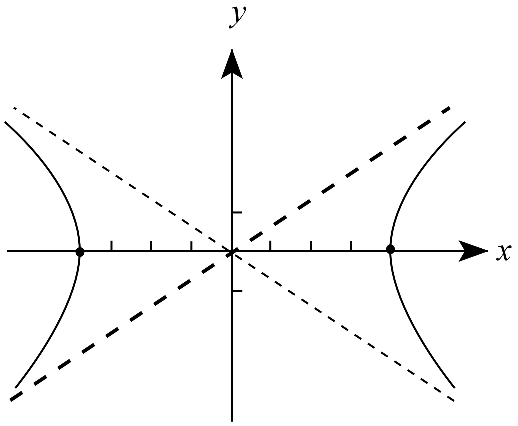
Now $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ is the equation of an ellipse.

(b) The parabola is symmetric with respect to the $y$-axis, and its equation is of the form $x^2 = cy$, $c$ a constant. Since the parabola passes through the point $(-3, 5)$, the equation of the parabola is satisfied by $x = -3,\ y = 5$. Thus, $(-3)^2 = c\,(5)$, and $c = 9/5$. The equation of the parabola is $x^2 = 9/5\,y$, or $x^2 = 4py$, with $4p = 9/5$, $p = 9/20$. This shows the focus of the parabola is $F(0, 9/20)$, and the directrix has equation $y = -9/20$.

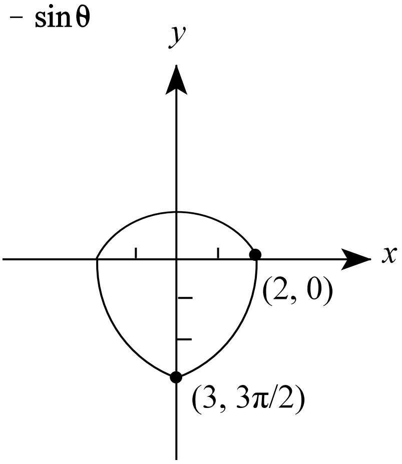


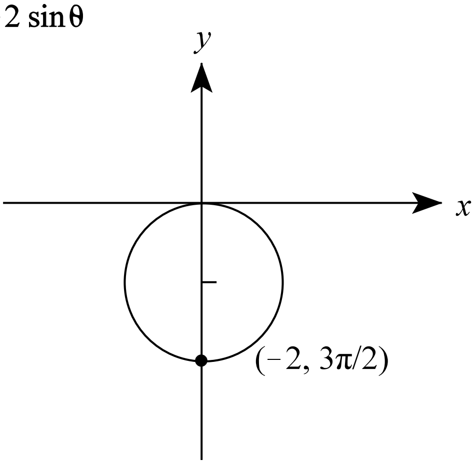
10. (a) The given equation of the hyperbola is of the form $x^2/a^2 - y^2/b^2 = 1$, with $a^2 = 16$, $b^2 = 4$. The vertices are $(\pm 4, 0)$, and the foci at $(\pm c, 0)$, where $c^2 = a^2 + b^2 = 20$, and $c = \sqrt{20} = 2\sqrt{5} \approx 4.47$. The asymptotes have equations $y = b/a\,(x) = \pm(2/4)x = \pm(1/2)x$.

(b)



11. (a) $r = 2 - \sin\theta$ (b) $r = -2\sin\theta$



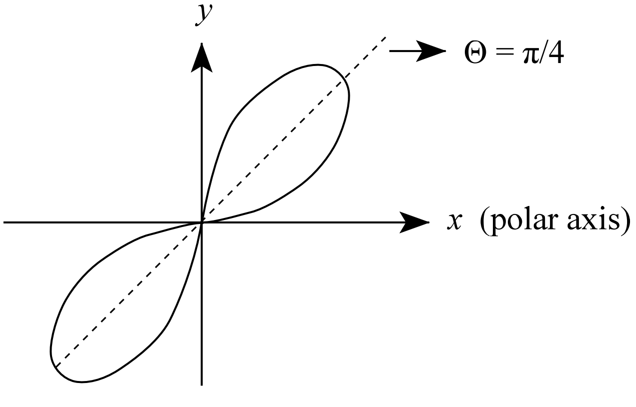


In (b), to change the equation $r = -2\sin\theta$ to rectangular coordinates, observe

$$r = -2\left(\frac{y}{r}\right) \Leftrightarrow r^2 = -2y \Leftrightarrow x^2 + y^2 = -2y$$

or $x^2 + y^2 + 2y = 0 \Leftrightarrow x^2 + (y+1)^2 = 1$. This is the rectangular coordinate equation of a circle with center at $(x = 0,\ y = -1)$, and radius 1.

12. (a) The graph of the lemniscate $r^2 = 2\sin 2\theta$ is roughly as sketched below.



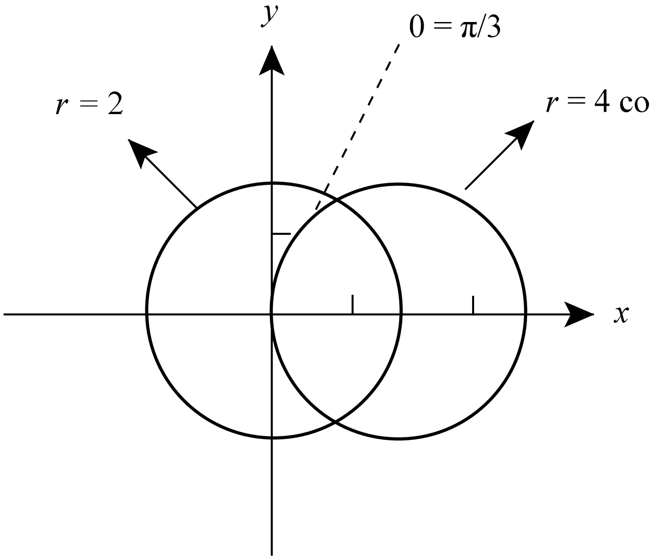
***Note:*** Since the equation $r^2 = 2\sin 2\theta$ isn't changed when $r$ is replaced by $-r$, the graph is symmetric with respect to the pole (the origin here). The area is therefore twice the area of the loop in the first quadrant, and is

$$2\cdot\int_0^{\pi/2}\frac{1}{2}r^2\,d\theta = 2\int_0^{\pi/2}\sin 2\,\theta\,d\theta$$

$$= 2\,(-\frac{1}{2}\cos 2\theta\,\Big|_0^{\pi/2} = -\cos\pi + \cos 0$$

$$= -(-1) + 1 = 2.$$

12. (b) The graphs of the circles $r = 2$ and $r = 4\cos\theta$ are depicted below.



The circles intersect where $4\cos\theta = 2 \Leftrightarrow \cos\theta = 1/2 \Leftrightarrow \theta = \pi/3$.

(i) The area inside the graphs of both circles is

$$2\cdot\frac{1}{2}\int_0^{\pi/3}2^2\,d\theta + 2\cdot\frac{1}{2}\int_{\pi/3}^{\pi/2}(4\cos\theta)^2\,d\theta$$

$$= \int_0^{\pi/3}4\,d\theta + \int_{\pi/3}^{\pi/2}16\cos^2\theta\,d\theta.$$

(ii) The area outside the graph of $r = 2$ and inside the graph of $r = 4\cos\theta$ is

$$2\cdot\frac{1}{2}\int_0^{\pi/3}\left[(4\cos\theta)^2 - 2^2\right]d\theta.$$

13. (a) (1) $y = 3$ is the equation of a plane parallel to the $x, z$ plane and 3 units to its right.

(2) $x = y$ is the equation of a plane perpendicular to the $x, y$ plane, and intersects the $x, y$ plane along the line $x = y$ in the $x, y$ plane.

(3) $x^2 + y^2 = 4$ is the equation of a right circular cylinder perpendicular to the $x, y$ plane. The cross section cut out of the cylinder by the $x,y$ plane is the circle $x^2 + y^2 = 4$ in the $x, y$ plane. The cylinder has radius 2, and its axis is the $z$-axis.

(4) $z = \sqrt{9 - x^2 - y^2} \Rightarrow x^2 + y^2 + z^2 = 9$, which is a sphere, with center at the origin and radius 3; so $z = \sqrt{9 - x^2 - y^2}$ is the equation of the top half of this sphere (a hemisphere) where $z \geq 0$.

(5) $xy = 0$ is satisfied by $x = 0$ or $y = 0$. Now in three-space, the graph of $x = 0$ is the $y, z$ plane, and the graph of $y = 0$ is the $x, z$ plane; thus the graph of $xy = 0$ is the union of the $y, z$ plane and the $x, z$ plane.

(b) (i) $\vec{a} = \langle 1 - 0,\ 3 - 0,\ 4 - 1 \rangle = \langle 1,\ 3,\ 3 \rangle$.

(ii) $\dfrac{x - 1}{1} = \dfrac{y - 3}{3} = \dfrac{z - 4}{3}$, or equally well, $\dfrac{x}{1} = \dfrac{y}{3} = \dfrac{z - 1}{3}$.

14. Recall the basic result

$$\vec{a} \cdot \vec{b} = |\vec{a}|\ |\vec{b}| \cos\theta$$

where $\theta$ is the angle between the vectors $\vec{a}$ and $\vec{b}$. From the preceding equation, we have

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\ |\vec{b}|} = \frac{\langle 1, 1, 0 \rangle \cdot \langle 0, 1, 1 \rangle}{\sqrt{1^2 + 1^2 + 0^2} \cdot \sqrt{0^2 + 1^2 + 1^2}}$$

$$= \frac{1(0) + (1)(1) + 0(1)}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}.$$

Hence, $\theta = \cos^{-1}\left(\dfrac{1}{2}\right) = \pi/3$.

15. Let $\vec{r}\,(t)$ denote the position vector from the origin $(0, 0, 0)$ to a general point $P\,(x, y, z)$ on the curve. Here,

$$\vec{r}\,(t) = x\vec{i} + y\vec{j} + z\vec{k} = (2 + \sin t)\,\vec{i} + (\cos t)\,\vec{j} + t\,\vec{k}$$

Clearly, the point $P\,(2, 1, 0)$ corresponds to $t = 0$; so you want to compute first $\vec{r}\,'(0)$.

***Note*** $\vec{r}\,'(t) = (\cos t)\,\vec{i} - (\sin t)\,\vec{j} + \vec{k}$ ; so $\vec{r}\,'(0) = \vec{i} + \vec{k}$, and $|\vec{r}\,'(0)| = \sqrt{2}$.

The desired unit tangent vector is $\dfrac{\vec{r}\,'(0)}{|\vec{r}\,'(0)|} = \dfrac{1}{\sqrt{2}}\,(\vec{i} + \vec{k})$.

16. (a) $$\vec{a} \text{ x } \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 3 & -1 \\ 2 & -5 & 6 \end{vmatrix} = \vec{i}\,(18 - 5) - \vec{j}\,(24 + 2) + \vec{k}\,(-20 - 6) = 13\,\vec{i} - 26\,\vec{j} - 26\,\vec{k}$$

(b) Since the plane whose equation we seek is parallel to the plane containing the vectors $\vec{a}$ and $\vec{b}$, normal vectors to these two planes have the same direction. Now recall that $\vec{a} \text{ x } \vec{b}$ is normal to the plane containing the vectors $\vec{a}$ and $\vec{b}$. Thus, as a normal vector to the plane containing the point $(2, -3, 2)$,we can take the vector $13\,\vec{i} - 26\,\vec{j} - 26\,\vec{k}$ computed in 16(a); equally well, we can use as the normal vector to this plane the vector $\vec{i} - 2\vec{j} - 2\,\vec{k}$. The equation of the plane through $(2, -3, 2)$ is thus $1\,(x - 2) - 2\,(y + 3) - 2\,(z - 2) = 0$, or equivalently, $x - 2y - 2z - 4 = 0$.