OHIO UNIVERSITY

MATH 163B CCE — SAMPLE EXAMINATION

Directions: Allow yourself three hours to work the problems. You will gain the most benefit from this test if you do not use your textbook or notes. You may, however, use your calculator. Work the problems neatly and carefully and use graph paper where needed.

Work all the problems, then check them with the answer key that follows the test. Total your points and carefully review any weak points before you apply for the supervised examination.

1. (5 Points) Answer "TRUE" or "FALSE" for each of the following.

(a) $f(x) = (-2)^x$ is an exponential function.

(b) The logarithmic function is defined for all real numbers x.

(c) $e^{\ln x} = x = \ln(e^x)$

(d) For a function f continuous over the interval $[a,b]$, the average value is equal to

$$\frac{1}{(b-a)}\int_b^a f(x)\,dx.$$

(e) $\ln(x+y) = \ln x + \ln y$.

2. (5 Points) Fill in the blanks (use capital letters) to make the following statements correct.

(a) $\log_b(x^n)$ is equal to ______________________ .

(b) The antiderivative of e^x is equal to ______________________ .

(c) If $f(x) = e^{x^2}$, then $f'(x) =$ ______________________ .

(d) If $F(x) = \int f(x)\,dx$, then $\int_a^b f(x)\,dx =$ ______________________ .

(e) The point where supply and demand curves intersect is called ________________ .

3. (5 Points) Find the derivatives of the following functions. Do not simplify the answers.

(a) $f(x) = xe^{-x} + \ln(x+1) + e^{x^2} + 3^x$

(b) $g(x) = \ln\{(x^2+9)^2 \cdot (x+1)\}$ (Hint: use first a property of the logarithmic function)

4. (5 Points) Find the antiderivative of each of the following functions.

(a) $f(x) = -3x^{-3} + \sqrt{x} + 2^x$

(b) $g(x) = \dfrac{1}{2x + 3} + e^{-2x + 3}$

5. (5 Points) Evaluate the definite integral. Show all work.

$$\int_0^1 x e^{x^2} dx$$

6. (5 Points) Evaluate the definite integral. Show all work.

$$\int_1^e x^2 \ln x \, dx$$

7. (10 Points) Shade the area bounded by the graphs of the functions $f(x) = x^2$ and $g(x) = 8 - x^2$. Next, compute this area.

8. (10 Points) The population of a country is growing exponentially at the rate of 2.5% per year. How many years does it take for it to double?

9. (10 Points) The demand equation for a product is: $p = \sqrt{81 - x}$, $0 < x < 10$.

(a) Express demand x as a function of price.

(b) Find the elasticity of demand when $p = 6$.

(c) Is the demand elastic, inelastic, or none?

10. (10 Points) Draw the graph of the demand equation: $p = 500 - 10x$ and the supply equation: $p = 0.5x^2 + 100$ for a certain product. Shade areas of consumer's surplus and producer's surplus, then compute both the these surpluses.

11. (10 Points)

(a) Solve the equation

$$5e^{x-3} = 4$$

(b) Graph the function

$$g(x) = 1 + \ln x$$

12. (10 Points)

(a) Compute $\ln\left(\frac{1}{e^2}\right)$

(b) The number of workers for a large company is given by $N(t) = \frac{1}{2}(t+2)^{2/3}$, where t is measured in years and $N(t)$ is measured in hundreds. Determine the average number of employees over the last six years.

13. (10 Points)

(a) The marginal revenue (in million dollars) is given by $R'(x) = 3x - x\,e^{-x^2}$ where x is the number of units (in thousands) manufactured and sold. If the revenue is zero when the level of production is zero, find the revenue when the level of production is 5000 units.

(b) Determine whether the improper integral $\int_{-\infty}^{\infty} \frac{8x}{(x^2+1)^{1/3}}\,dx$ converges or diverges. If it converges, then find its value.

Bonus Problem (5 Points)

Suppose that $10,000 is invested at an annual rate of 12% compounded bimonthly. What is the future value y of the investment after two years?

ANSWER KEY TO SAMPLE EXAMINATION

1. (a) F (b) F (c) T (d) T (e) F

2. (a) $n \log_b x$

(b) e^x

(c) $f'(x) = 2x\,e^{x^2}$

(d) $F(b) - F(a)$

(e) Equilibrium point

3. (a) $$f'(x) = 1 \cdot e^{-x} + xe^{-x} \cdot (-1) + \frac{1}{x+1} + 2xe^{x^2} + 3^x \ln 3$$

(b) Write: $f(x) = 2\ln(x^2+9) + \ln(x+1)$

$$f'(x) = \frac{(2x)(2)}{x^2+9} + \frac{1}{x+1}$$

4. (a) $$\frac{3}{2}x^{-2} + \frac{2}{3}x^{\frac{3}{2}} + \frac{2^x}{\ln 2} + C$$

(b) $$\frac{1}{2}\ln(2x+3) - \frac{1}{2}e^{-2x+3} + C$$

5. Using substitution method $u = x^2$ we get

$$\int xe^{x^2}\,dx = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u = \frac{e^{x^2}}{2}$$

$$\int_0^1 xe^{x^2} = \frac{1}{2}(e^1 - e^0) = \frac{1}{2}(e-1)$$

6. Using "integration by parts" method:

$$\frac{x^3}{3}\ln x - \int \frac{x^3}{3} \cdot \frac{1}{x}\,dx$$

$$= \frac{x^3 \ln x}{3} - \int \frac{x^2}{3}\,dx$$

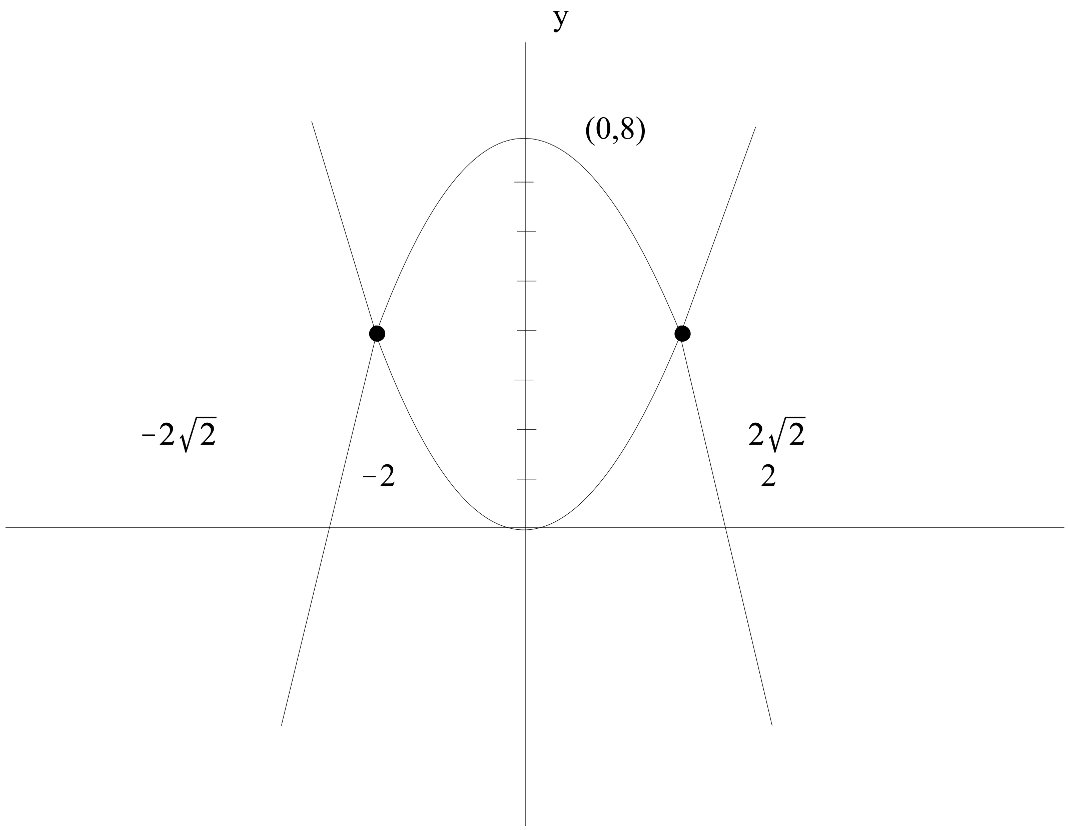
$$= \frac{x^3 \ln x}{3} - \frac{x^3}{9}$$

Now $$\int_1^e x^2 \ln x = \left.\frac{x^3 \ln x}{3} - \frac{x^3}{9}\right]_1^e$$

$$= \left(\frac{e^3}{3} - \frac{e^3}{9}\right) - \left(-\frac{1}{9}\right)$$

$$= \frac{2e^3+1}{9}$$

7.



Using symmetry:

$$\text{Area} = 2\int_0^2 ((8 - x^2) - x^2)\,dx$$
$$= 2\int_0^2 (8 - 2x^2)\,dx$$
$$= 2\left(\frac{32}{3}\right) = \frac{64}{3}$$

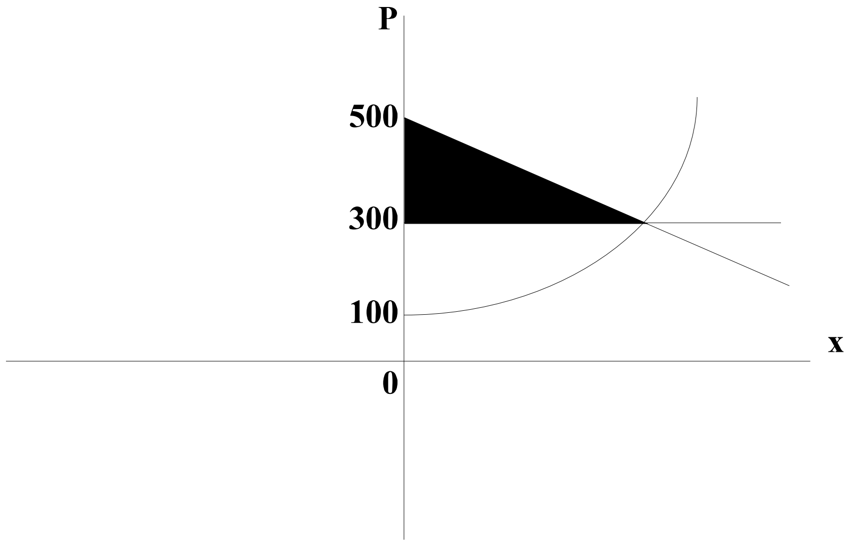
8. Using the formula $P = P_0 e^{at}$ get $t = \dfrac{\ln 2}{.025} \approx 27.72$ years

9. (a) $x = 81 - p^2$, so $f(p) = 81 - p^2$

(b) Elasticity of demand $E(p) = \dfrac{pf'(p)}{f(p)}$. Compute $E(6) = \dfrac{6 \cdot f'(6)}{f(6)} = -\left(\dfrac{72}{45}\right)$

(c) Demand is elastic.

10. Find equilibrium point $x = 20$, $p = 300$. For consumer surplus, compute area between the curves $p = 500 - 10x$ and $p = 300$



11. (a) $x = \ln\left(\frac{4}{5}\right) + 3$

(b) Draw the graph of $\ln x$ and raise it vertically by one unit.

12. (a) $\ln 1 - 2 \ln e = -2$

(b) Compute the integral $\frac{1}{(6-0)}\left(\frac{1}{2}\int_0^6 (t+2)^{2/3}\,dt\right) = \frac{1}{12}\int_0^6 (t+2)^{2/3}\,dt$

≈ 1.44

13. (a) $R(x) = \frac{3}{2}x^2 + \frac{1}{2}e^{-x^2}$

Compute $R(5000) = \frac{3}{2}(5000)^2 + \frac{1}{2}e^{-(5000)^2}$

(b) Diverges.

Bonus Problem.

Using the formula, compute: $A = 10{,}000\left(1 + \frac{.12}{6}\right)^{(6)(2)}$