



MATH 113 CCE — SAMPLE EXAMINATION

The problems in this sample are intended to help you determine if you are ready to take the supervised examination. You will gain the most benefit from it if you set yourself a time limit of three hours and work the problems without using a calculator, books, or notes. Be sure to write out all the steps in the problem; you will be **required** to show your work to receive credit for the problems on the supervised examination. Check your answers with the key that follows, then review the relevant text sections for any problems you miss.

Solve the following equations and inequalities.

1. $4(x-2) - 3(x-1) = 2(x+6)$

2. $\frac{3a-1}{4} + \frac{a-2}{3} - \frac{a-1}{5} = \frac{21}{20}$

3. Solve the following equation for the variable S .

$$a(S-t) + b(t+S) = 7$$

4. $\frac{3-x}{6} + \frac{x+2}{7} \leq 1$

5. $|2-3x| < 5$

6. (a) Simplify by writing as a single fraction with no negative exponents.

$$(x + x^{-1})^{-1}$$

(b) Solve the equation.

$$\sqrt[3]{2x+3} = -3$$

7. Find the product $(x^{2a} + 6)(x^{2a} - 4)$.

8. Factor *completely* the following.

(a) $2x^3 + 8x^2 - 3x - 12$

(b) $6x^2 + 5x - 6$

9. Perform the indicated operations and write result in simplest form.

$$\frac{2x^2 + 3x}{2x^3 - 10x^2} \cdot \frac{x^2 - 8x + 15}{3x^3 - 27x} \div \frac{14x + 21}{x^2 - 6x - 27}$$

10. Combine into one fraction written in simplest form.

$$\frac{2}{y^2 + 6y - 16} - \frac{4}{y + 8} - \frac{3}{y - 2}$$

11. Perform the indicated “long” division. $(x^4 + 2x^3 - 5x^2 - 4x + 9) \div (x^2 - 2)$

12. Solve for x .

$$\sqrt{x + 4} = \sqrt{x - 1} + 1$$

13. (a) Write $\sqrt{-32}$ in terms of i and simplify.

(b) Multiply $(2 - 7i)(4 + 5i)$.

Solve using the quadratic formula.

14. $2n^2 - 8n = -3$

15. $x^2 + 11 = -6x$

16. $\frac{-4}{x + 1} + \frac{5}{3} = \frac{2}{x}$

17. Find the equation of a line passing through the point $(-1, 3)$ and perpendicular to the line $2x - y = 4$.

18. Solve the system of equations

$$\frac{1}{2}x - \frac{1}{3}y = 12$$

$$\frac{3}{4}x + \frac{2}{3}y = 4$$

19. Solve the quadratic inequality $3x^2 + 13x - 10 \leq 0$.

20. The formula $D = \frac{n(n - 3)}{2}$ gives the number of diagonals, D , in a polygon having n sides. Find the number of sides of a polygon having 44 diagonals.

21. (a) Rationalize the denominator

$$\frac{3}{\sqrt{2} + i}$$

- (b) Write in scientific notation the following:

(i) .0000345

(ii) 3450000.

22. Evaluate $(2^{-3} + 3^2)^{-1}$.
23. Solve the system of equations.

$$\begin{cases} x - 2y + 3w = 7 \\ 2x + y + 5w = 17 \\ 3x - 4y - 2w = 1 \end{cases}$$

ANSWER KEY FOR SAMPLE EXAMINATION

1. $4(x-2) - 3(x-1) = 2(x+6)$
 $4x - 8 - 3x + 3 = 2x + 12$
 $x - 5 = 2x + 12$
 $-x = 17$
 $x = -17$

2. $\frac{3a-1}{4} + \frac{a-2}{3} - \frac{a-1}{5} = \frac{21}{20}$ LCD = 60

$$45a - 15 + 20a - 40 - 12a + 12 = 63$$

$$53a - 43 = 63$$

$$53a = 106$$

$$a = 2$$

3. $a(S-t) + b(t+S) = 7$
 $aS - at + bt + bS = 7$
 $S(a+b) = 7 + at - bt$
 $S = \frac{7 + at - bt}{(a+b)}$

4. $\frac{3-x}{6} + \frac{x+2}{7} \leq 1$
 $21 - 7x + 6x + 12 \leq 42$
 $-x + 33 \leq 42$
 $-x \leq 9$
 $x \geq -9$

5. $|2-3x| < 5$ $3x < 7$
 $-5 < 2-3x < 5$ $x < 7/3$
 $-7 < -3x < 3$ $3x > -3$
 $7 > 3x > -3$ $x > -1$

Answer: $-1 < x < 7/3$

6. (a) $(x + x^{-1})^{-1} = \left(x + \frac{1}{x}\right)^{-1} = \left(\frac{x^2 + 1}{x}\right)^{-1} = \frac{x}{x^2 + 1}$

(b) $\sqrt[3]{2x+3} = -3; 2x+3 = -27; 2x = -30; x = -15$

7. $(x^{2a} + 6)(x^{2a} - 4) = x^{2a+2a} + 6x^{2a} - 4x^{2a} - 24 = x^{4a} + 2x^{2a} - 24$

8. (a) $2x^3 + 8x^2 - 3x - 12 = 2x^2(x+4) - 3(x+4) = (2x^2 - 3)(x+4)$

(b) $6x^2 + 5x - 6 = (2x+3)(3x-2)$

9. $\frac{\cancel{x}(2\cancel{x}+3)}{2x^2(\cancel{x}-5)} \cdot \frac{(\cancel{x}-5)(\cancel{x}-3)}{3x(\cancel{x}-3)(x+3)} \cdot \frac{(x-9)(x+3)}{7(2x+3)}$

Divide out common factors. Write result.

$$= \frac{x-9}{42x^2}$$

10. $\frac{2}{(y+8)(y-2)} - \frac{4}{y+8} - \frac{3}{y-2}$ Put fractions over LCD of $(y+8)(y-2)$.

$$\frac{2 - 4(y-2) - 3(y+8)}{(y+8)(y-2)} = \frac{2 - 4y + 8 - 3y - 24}{(y+8)(y-2)} = \frac{-7y - 14}{(y+8)(y-2)} = \frac{-7(y+2)}{(y+8)(y-2)}$$

11.
$$\begin{array}{r} x^2 + 2x - 3 \\ x^2 - 2 \overline{) (x^4 + 2x^3 - 5x^2 - 4x + 9)} \\ \underline{x^4 - 2x^2} \\ 2x^3 - 3x^2 - 4x \\ \underline{2x^3 - 4x} \\ -3x^2 + 9 \\ \underline{-3x^2 + 6} \\ 3 \text{ remainder} \end{array}$$

12. $\sqrt{x+4} = \sqrt{x-1} + 1$ square *each* side

$$x+4 = (x-1) + 2\sqrt{x-1} + 1$$

$$4 = 2\sqrt{x-1}$$

$$2 = \sqrt{x-1}$$

$$4 = x-1$$

$$x = 5$$

13. (a) $\sqrt{-32} = \sqrt{32}i = 4\sqrt{2}i$

(b) $(2 - 7i)(4 + 5i) = 8 - 28i + 10i - 35(i^2)$
 $= 8 - 18i - 35(-1) = 8 - 18i + 35$
 $= 43 - 18i$

14. $2n^2 - 8n + 3 = 0$

$$n = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{4} = \frac{8 \pm \sqrt{64 - 24}}{4}$$

$$n = \frac{8 \pm \sqrt{40}}{4} = \frac{8 \pm 2\sqrt{10}}{4} = \frac{4 \pm \sqrt{10}}{2}$$

15. $x^2 + 6x + 11 = 0$

$$x = \frac{-6 \pm \sqrt{36 - 44}}{2} = \frac{-6 \pm \sqrt{-8}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{2}i}{2} = -3 \pm \sqrt{2}i$$

16. $\frac{-4}{x+1} + \frac{5}{3} = \frac{2}{x}$

$$\frac{-12 + 5x + 5}{3(x+1)} = \frac{2}{x}$$

$$\frac{5x - 7}{3(x+1)} = \frac{2}{x}$$

$$5x^2 - 7x = 6x + 6$$

$$*5x^2 - 13x - 6 = 0$$

$$(5x + 2)(x - 3) = 0$$

$$x = -2/5, x = 3$$

**Quadratic formula could be used.*

17. $2x - y = 4$ is solved for y
 $y = +2x - 4$ slope is $(+2)$.

Perpendicular line has slope $\left(\frac{-1}{2}\right)$

$$y - (3) = \frac{-1}{2} (x - (-1))$$

$$2y - 6 = -x - 1$$

$x + 2y = 5$ is perpendicular line equation.

18. $\frac{1}{2}x - \frac{1}{3}y = 12 \rightarrow 3x - 2y = 72$

$$\frac{3}{4}x + \frac{2}{3}y = 4 \rightarrow 9x + 8y = 48$$

$$9x - 6y = 216$$

$$\underline{9x + 8y = 48}$$

$$-14y = 168$$

$$y = -12$$

$$\frac{1}{2}x - \frac{1}{3}(-12) = 12, \frac{1}{2}x + 4 = 12$$

$$\frac{1}{2}x = 8$$

$$x = 16$$

19. $3x^2 + 13x - 10 \leq 0$

$$(3x - 2)(x + 5) \leq 0$$

Critical values are $x = 2/3, x = -5$

If $x > 2/3$ function is positive so $x \leq 2/3$.

If $x < -5$ function is positive so $x \geq -5$.

Solution $-5 \leq x \leq 2/3$

$$20. \quad D = \frac{n(n-3)}{2}$$

If $D = 44$ then

$$44 = \frac{n(n-3)}{2}$$

$$88 = n^2 - 3n$$

$$n^2 - 3n - 88 = 0$$

$$(n-11)(n+8) = 0$$

$n = 11$ sides

$n = -8$ impossible

$$21. \quad (a) \quad \frac{3}{\sqrt{2}+i} = \frac{3}{\sqrt{2}+i} \cdot \frac{\sqrt{2}-i}{\sqrt{2}-i} = \frac{3(\sqrt{2}-i)}{2-i^2} = \frac{3(\sqrt{2}-i)}{2-(-1)} = \frac{3(\sqrt{2}-i)}{2+1} = \frac{3(\sqrt{2}-i)}{3} = \sqrt{2}-i$$

$$(b) \quad .0000345 = 3.45 \times 10^{-5}$$

$$3450000 = 3.45 \times 10^6$$

$$22. \quad (2^{-3} + 3^2)^{-1} = \left(\frac{1}{2^3} + 3^2\right)^{-1} = \left(\frac{1}{8} + 9\right)^{-1} = \left(\frac{1}{8} + \frac{72}{8}\right)^{-1} = \left(\frac{73}{8}\right)^{-1} = \frac{8}{73}$$

$$23. \quad \begin{array}{ll} \textcircled{A} & x - 2y + 3w = 7 \\ \textcircled{B} & 2x + y + 5w = 17 \\ \textcircled{C} & 3x - 4y - 2w = 1 \end{array} \quad \begin{array}{l} \text{Eliminate } y \text{ from } \textcircled{A} \text{ and } \textcircled{B}. \\ x - 2y + 3w = 7 \\ \underline{4x + 2y + 10w = 34} \\ 5x + 13w = 41 \end{array}$$

Eliminate y from \textcircled{A} and \textcircled{C} .

$$\begin{array}{r} 2x - 4y + 6w = 14 \\ 3x - 4y - 2w = 1 \\ \hline -x \quad + 8w = 13 \end{array} \quad \begin{array}{r} 5x + 13w = 41 \\ -5x + 40w = 65 \\ \hline 53w = 106 \\ w = 2 \end{array}$$

$$\text{If } w = 2 \text{ then } -x + 8(2) = 13$$

$$-x + 16 = 13$$

$$x = 3$$

$$\text{Finally, if } x = 3 \text{ and } w = 2 \text{ then } 3 - 2y + 3(2) = 7$$

$$3 - 2y + 6 = 7$$

$$-2y = -2$$

$$y = 1$$

Answer $x = 3, y = 1, w = 2$