Resilient Modulus Prediction Model for Fine-Grained Soils in Ohio: Preliminary Study

by

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ABSTRACT

Resilient modulus of subgrade soil is one of the key material properties that are required for the mechanistic-empirical design/analysis of multi-layered flexible pavement system. A study was initiated at Ohio University to examine the resilient modulus of fine-grained subgrade soils commonly found in Ohio. The current standard method was first examined to understand the stress conditions the test procedure induces on the soil specimen. Then, five stress models were briefly described and evaluated in light of recent laboratory test results on A-4 soil samples recovered from a highway project site in the northeastern Ohio. The outcome of the initial study indicated the hyperbolic model may be the most promising model. Preliminary correlations were established between the hyperbolic model constants and the test specimens' basic properties. Additional test results will be needed to develop a resilient modulus prediction model for the A-4 Ohio soils at higher statistical confidence levels. Also, the modeling efforts should extend to address the other subgrade soil types commonly found in Ohio.

INTRODUCTION

Resilient modulus of subgrade soil is one of the key material properties that are required for the mechanistic-empirical (M-E) design/analysis of multi-layered flexible pavement system. At the highest level (Level 1), the resilient modulus of the subgrade soil must be measured in the laboratory, using the representative soil samples recovered from the project site. The current standard laboratory practice is to perform the resilient modulus test according to the procedures described in the AASHTO T-294 or SHRP P-46 test protocol.

The test set-up used in the resilient modulus (RM) test is basically the same as that for the conventional triaxial compression (CTC) test. A cylindrical soil specimen, encased in a flexible membrane, is subjected to an axial loading applied by a piston, while the all-around chamber pressure acts on it. The only difference between the CTC and RM tests is that the axial load is applied as a series of 1-Hz pulse load during RM test while it is increased monotonically during the CTC test. In the RM test, miniature LVDT's mounted near the test specimen measure the average recoverable axial strain.

The resilient modulus (M_R) is calculated from the test measurements by:

$$M_R = \sigma_d/\varepsilon_r \quad (1)$$

where σ_d = deviatoric stress; and ε_r = recoverable axial strain.

Most fine-grained soils exhibit a concave-upward bilinear response curve, when their RM test data are plotted in terms of M_R versus σ_d. Breakpoint resilient modulus (M_{Ri}) is the resilient modulus located at the point where two linear curves meet.

Principal stresses involved in these test methods are:

$$\sigma_1 = \sigma_d + \sigma_c \quad ; \qquad \text{and} \qquad \sigma_2 = \sigma_3 = \sigma_c \quad (2 \ \& \ 3)$$

where σ_1 = major principal stress; σ_d = deviatoric stress (= axial stress applied by the piston); σ_c = chamber pressure (or confining stress); and σ_2 or σ_3 = minor principal stress.

The stress path followed during these tests can be represented by the parameters p and q:

$$p = \frac{1}{2}(\sigma_1 + \sigma_3) \quad ; \qquad \text{and} \quad q = \frac{1}{2}(\sigma_1 - \sigma_3) \quad (4 \ \& \ 5)$$

Octahedral shear stress (τ_{oct}) and normal stress (σ_{oct}) are defined as:

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \ ; \qquad \text{and} \quad \sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \quad (6 \ \& \ 7)$$

Applications of Equations 2 and 3 to Equations 4 through 7 result in:

$$p = \frac{\sigma_d}{2} + \sigma_3 = q + \sigma_3; \quad \text{and} \quad q = \frac{\sigma_d}{2} \quad (4' \ \& \ 5')$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_d)^2 + (\sigma_d)^2} = \frac{\sqrt{2}}{3}\sigma_d\,; \quad \text{and} \quad \sigma_{oct} = \frac{1}{3}(\sigma_d + 3\sigma_c) \qquad (6' \,\&\, 7')$$

Table 1 lists the four stress parameter (p, q, τ_{oct}, σ_{oct}) values calculated for the load sequences the soil specimen goes through during the standard RM test for fine-grained soil specimens. The chamber pressure is zero during Load Sequence Nos. 11 through 15. So, during these load sequences, test results reflect the effect of the deviatoric stress only.

TABLE 1 Load Sequence Applied During RM Test

σ_c (psi)	Seq. No.	σ_d (psi)	No. of Cycles	p (psi)	q (psi)	τ_{oct} (psi)	σ_{oct} (psi)
6.0 *	0	4.0	Up to 500	8.0	2.0	1.89	7.33
6.0	1	2.0	100	7.0	1.0	0.94	6.67
	2	4.0	100	8.0	2.0	1.89	7.33
	3	6.0	100	9.0	3.0	2.83	8.00
	4	8.0	100	10.0	4.0	3.77	8.67
	5	10.0	100	11.0	5.0	4.71	9.33
3.0	6	2.0	100	4.0	1.0	0.94	3.67
	7	4.0	100	5.0	2.0	1.89	4.33
	8	6.0	100	6.0	3.0	2.83	5.00
	9	8.0	100	7.0	4.0	3.77	5.67
	10	10.0	100	8.0	5.0	4.71	6.33
0.0	11	2.0	100	1.0	1.0	0.94	0.67
	12	4.0	100	2.0	2.0	1.89	1.33
	13	6.0	100	3.0	3.0	2.83	2.00
	14	8.0	100	4.0	4.0	3.77	2.67
	15	10.0	100	5.0	5.0	4.71	3.33

[Note] * Initial conditioning load cycles.

Figure 1 shows the p-q diagram for the RM test. During the five load sequences under each chamber pressure level, both p and q values are increased by 4 psi. The stress path makes a 45° angle with the p-axis to make sure that the stress state does not approach the yield surface.

Figure 2 shows the changes in the octahedral stresses during the RM test. The octahedral shear stress (τ_{oct}) goes through the same cycle three times, with its value changing between 0.94 and 4.71 psi. The Octahedral normal stress (σ_{oct}) goes through a gradual stepwise decline. The ratio between the Octahedral stresses (σ_{oct}/τ_{oct}) remains a constant (at about 0.7) during Load Sequence Nos. 11 through 15, because the chamber pressure is zero.

LABORATORY TEST PROGRAM

In the current study, a laboratory test program was initiated to try to measure according to the SHRP P-46 test protocol the resilient modulus (M_R) of six subgrade soil samples taken from a project site on Rt. 30 in Wayne County (near Akron), Ohio. Table 2 summarizes the basic properties of these soil samples. Tables 3 and 4 present the test data. Each soil sample was dried, pulverized, and then recompacted inside a split mold, using the static compaction method, to produce the test specimen.

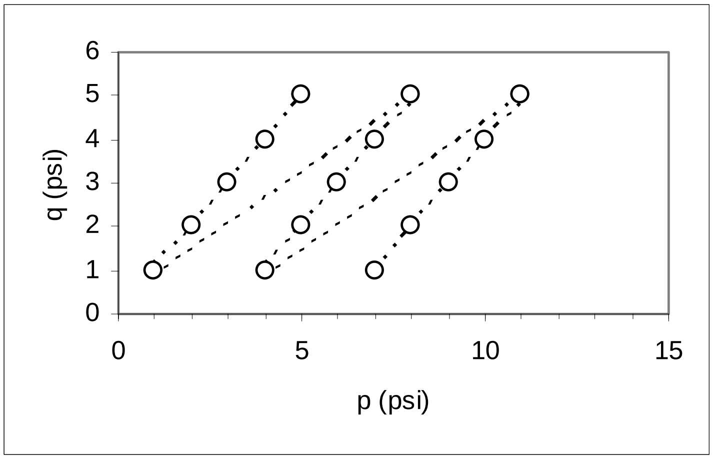

FIGURE 1 p-q Diagram (Stress Path) of RM Test

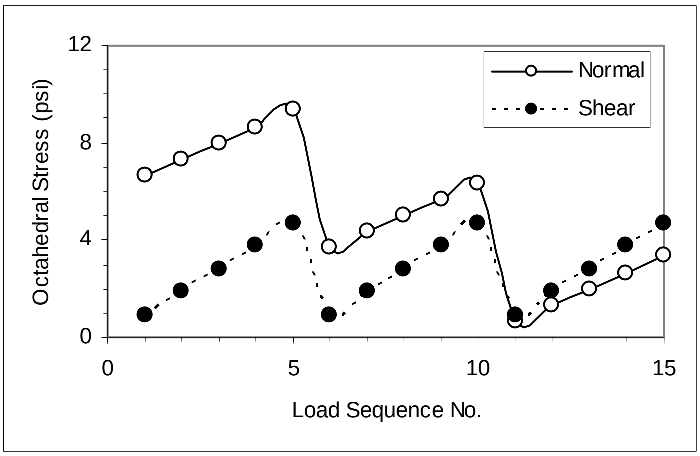

FIGURE 2 Octahedral Stress Changes During RM Test

TABLE 2 Basic Properties of Subgrade Soil Samples

Property	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6
Location	West Bound Sta. 885+00	East Bound Sta. 876+60	West Bound Sta. 876+60	East Bound Sta. 884+00	East Bound Sta. 884+00	East Bound Sta. 663+50
S200 (%)	46.9	38.2	39.3	41.7	48.9	41.0
LL (%)	26.9	26.7	25.9	26.0	24.4	23.3
PI (%)	9.9	9.4	8.2	7.3	8.0	6.2
OMC (%)	12.1	12.8	14.4	14.5	13.3	11.8
$\gamma_{d\text{-}max}$ (pcf)	117.5	118.0	117.5	116.0	117.5	121.0
Type	A-4	A-4	A-4	A-4	A-4	A-4

TABLE 3 RM Test Results for Sample Nos. 1 and 2

σ_c (psi)	Sample 1 w = 13.1%; γ_d = 115.0 pcf			Sample 2 (Test A) w = 12.1%; γ_d = 115.4 pcf			Sample 2 (Test B) w = 13.2%; γ_d = 116.7 pcf		
	σ_d (psi)	ε_r (%)	M_R (ksi)	σ_d (psi)	ε_r (%)	M_R (ksi)	σ_d (psi)	ε_r (%)	M_R (ksi)
6.0	0.60	0.017	3.51	1.50	0.010	15.43	1.60	0.022	7.44
	2.54	0.073	3.50	6.38	0.064	10.03	5.28	0.105	5.04
	4.62	0.124	3.71	9.22	0.101	9.18	8.52	0.164	5.18
	6.69	0.178	3.76	16.73	0.211	7.94	11.32	0.221	5.13
	8.53	0.225	3.79	15.09	0.188	8.03	14.26	0.286	4.99
3.0	0.31	0.004	8.29	3.12	0.027	11.78	1.62	0.026	6.32
	2.40	0.060	4.00	5.82	0.068	8.51	5.06	0.109	4.66
	4.14	0.108	3.84	9.06	0.112	8.11	8.15	0.171	4.77
	5.86	0.156	3.75	12.00	0.151	7.93	10.96	0.226	4.85
	7.59	0.199	3.82	15.04	0.193	7.78	13.95	0.292	4.77
0.0	0.26	0.003	7.77	2.95	0.029	10.06	1.56	0.026	6.13
	2.10	0.053	3.99	5.75	0.078	7.59	4.86	0.107	4.55
	3.86	0.101	3.81	8.89	0.120	7.43	7.77	0.171	4.54
	5.44	0.144	3.77	11.81	0.163	7.26	10.56	0.231	4.57
	7.22	0.189	3.82	14.89	0.208	7.17	13.68	0.298	4.59

TABLE 4 RM Test Results for Sample Nos. 3 and 5

σ_c (psi)	Sample 3 (Test A) w = 15.1%; γ_d = 119.8 pcf			Sample 3 (Test B) w = 12.1%; γ_d = 113.9 pcf			Sample 5 w = 12.5%; γ_d = 111.7 pcf		
	σ_d (psi)	ε_r (%)	M_R (ksi)	σ_d (psi)	ε_r (%)	M_R (ksi)	σ_d (psi)	ε_r (%)	M_R (ksi)
6.0	1.39	0.029	4.80	2.76	0.028	9.91	1.70	0.026	6.45
	4.01	0.107	3.74	5.80	0.070	8.27	4.28	0.085	5.06
	6.21	0.174	3.57	8.37	0.116	7.23	6.67	0.145	4.59
	8.49	0.253	3.36	10.90	0.169	6.45	9.35	0.184	5.09
	11.08	0.318	3.49	13.53	0.214	6.33	11.94	0.233	5.12
4.0	1.69	0.031	5.41	2.78	0.039	7.08	1.81	0.026	6.96
	3.72	0.104	3.58	5.48	0.075	7.26	4.32	0.079	5.47
	5.88	0.185	3.18	7.96	0.122	6.52	6.69	0.130	5.13
	8.54	0.265	3.22	10.74	0.172	6.25	9.45	0.184	5.14
	11.17	0.333	3.35	13.60	0.221	6.14	12.22	0.234	5.22
2.0	1.68	0.033	5.12	2.56	0.031	8.14	1.74	0.025	7.02
	3.62	0.106	3.40	5.25	0.084	6.28	4.29	0.080	5.40
	5.75	0.191	3.01	7.66	0.134	5.71	6.64	0.130	5.10
	8.38	0.271	3.09	10.47	0.189	5.54	9.39	0.184	5.10
	11.19	0.341	3.28	13.49	0.242	5.57	12.34	0.235	5.25

Figures 3 through 5 were produced based on the data from Sample 3 (Test A). Figure 3 shows fluctuations of the resilient modulus during the entire load sequences. The bilinear nature of the relationship between the resilient modulus and deviatoric stress is appearing three times. Figure 4 plots the resilient modulus against the deviatoric stress. The bilinear trend is shown clearly in the plot. Figure 5 examines the relationship between the resilient modulus and the Octahedral stresses. Here, the plot correlates the resilient modulus and the Octahedral stress ratio (σ_{oct}/τ_{oct}) in the logarithmic scale. Despite some scattering, the emerging relationship appears to be nearly linear.

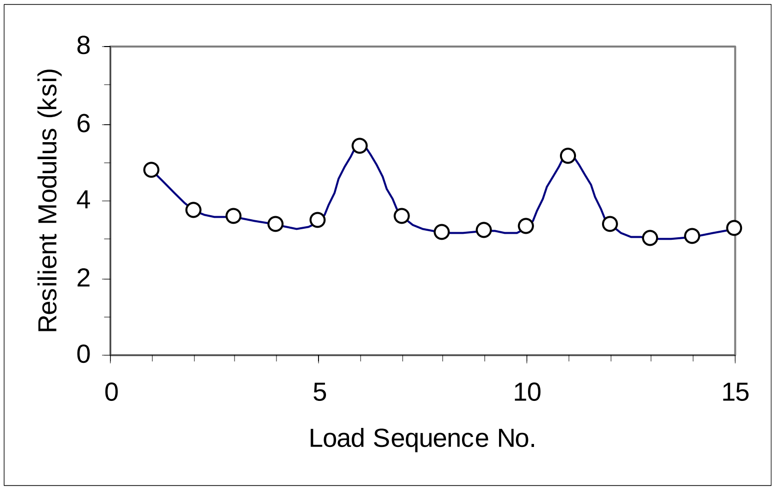

FIGURE 3 Fluctuations of M_R During RM Test

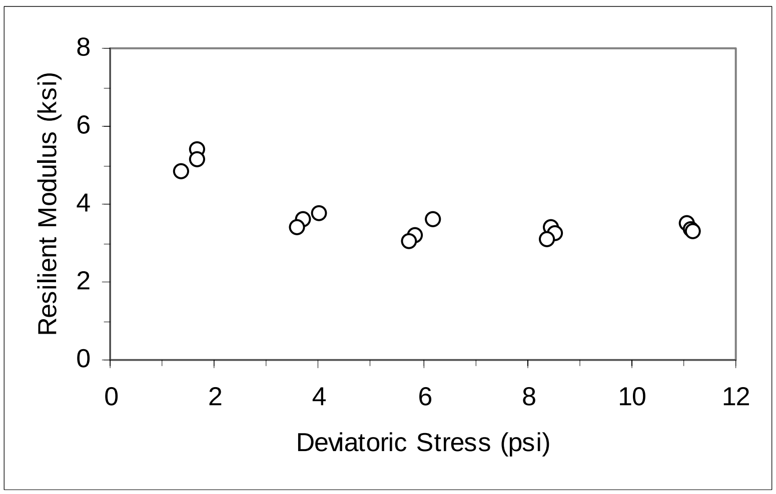

FIGURE 4 Plot of Resilient Modulus vs. Deviatoric Stress

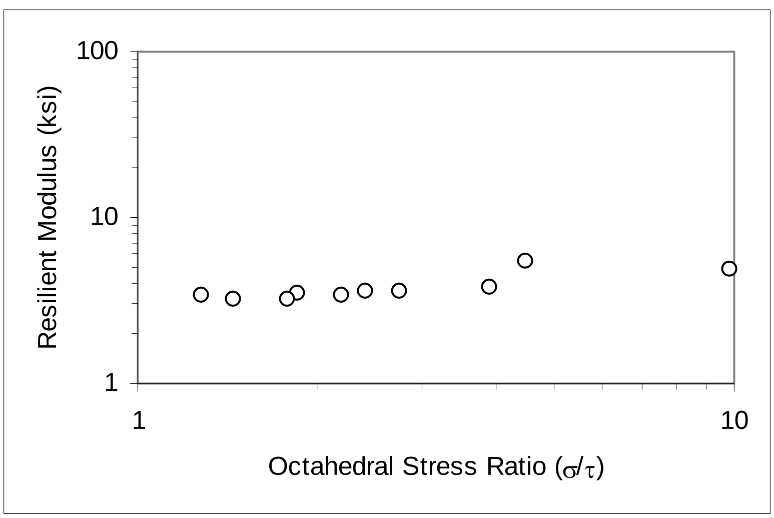

FIGURE 5 Plot of Log (M_R) vs. Log (σ_{oct}/τ_{oct})

MODELING OF RM BEHAVIORS OF FINE-GRAINED SOILS

At Level 2 or 3 of the M-E design procedure, a prediction model may be used to estimate the resilient modulus of subgrade soil below Level 1. Thus, it is important that a reliable model is identified. There have been a number of models proposed by other researchers for estimating the resilient modulus of fine-grained soils. The most basic model used in conjunction with the RM testing of fine-grained soils is a power model:

$M_R = K(\sigma_d)^n$ where K, n = model constants (8)

However, the power model cannot represent the bilinear relationship between the resilient modulus and deviatoric stress. The n value of 0 leads to the 0-th order relationship ($M_R = K$). A small negative value for n leads to a slightly nonlinear concave upward curve with no apparent break point.

A bilinear model has been developed by Dingqing and Selig (1994) to embrace the concept of the breakpoint resilient modulus:

$M_R = K_1 + K_2\sigma_d$ for $\sigma_d < \sigma_{di}$ (9.a)
$= K_3 + K_4\sigma_d$ for $\sigma_d > \sigma_{di}$ (9.b)

where K_1, K_2, K_3, and K_4 = model constants (K_1 and K_3 always positive; K_2 always negative; K_4 occasionally negative); and σ_{di} = breakpoint deviator stress.

A hyperbolic model was proposed by Drumm et al. (1991) for estimating the resilient modulus of fine-grained soils found in Tennessee:

$$M_R = \frac{K + n\sigma_d}{\sigma_d}$$ where K, n = model constants (10)

A semi-log model was proposed by Fredlund et al. (1977), who examined the resilient responses of a glacial till material:

$\text{Log}(M_R) = K - n\,\sigma_d$ where K, n = model constants (11)

A log-log model is presented in the SHRP P-46 test protocol as a means to plot the test data. This model, given by Eq. 12, is mathematically almost equivalent to the power model.

$\text{Log}(M_R) = K + n\,\text{Log}(\sigma_d)$ where K, n = model constants (12)

With the finding made in the previous section, an additional model (i.e., Octahedral model) may be worthy of evaluation:

$$Log(M_R) = K + n \cdot Log\left(\frac{\sigma_{oct}}{\tau_{oct}}\right)$$ where K, n = model constants (13)

Unlike all of the above models, the Octahedral model incorporates the effects of both deviatoric stress and confining stress. When the confining stress is set equal to zero, Eq. 13 cannot express M_R as a function of σ_d. In order to overcome this problem, Eq. 13 is modified to:

$$Log\left(M_R\right) = K' + n' \cdot Log\left[\frac{\sigma_{oct}}{\left(\tau_{oct}\right)^2}\right] \quad \text{where } K', n' = \text{model constants} \tag{13'}$$

EVALUATION OF MODELS

Table 5 summarizes the results of the data analysis performed for each candidate model, which include model constant values and the coefficient of determination (r^2) value. Comparing the overall average r^2 values, the hyperbolic model was considered to be the best model, followed by the Octahedral stress model and the bilinear model. The semi-log model was the least successful in fitting to the experimental RM test data.

For the bilinear model, the breakpoint resilient modulus (M_{Ri}) was determined to be 3.78 ksi (Sample 1), 8.21 ksi (Sample 2A), 4.80 ksi (Sample 2B), 3.17 ksi (Sample 3A), 6.35 ksi (Sample 3B), and 4.87 ksi (Sample 5). The breakpoint deviatoric stress (σ_{di}) was 2.39 psi (Sample 1), 6.28 psi (Sample 2A), 5.00 psi (Sample 2B), 4.45 psi (Sample 3A), 8.43 psi (Sample 3B), and 5.05 psi (Sample 5). The axial strain corresponding to the breakpoint (ε_i) was 0.06% (Sample 1), 0.07% (Sample 2A), 0.10% (Sample 2B), 0.13% (Samples 3A, 3B), and 0.10% (Sample 5).

TABLE 5 Evaluation of Stress Models

	Power Model – Eq. 8			Bilinear Model – Eq. 9					
	K	n	r^2	K_1	K_2	$(r_1)^2$	K_3	K_4	$(r_2)^2$
Sample 1	5.464	– 0.228	0.840	8.574	– 2.010	0.982	3.775	0.002	0.006
Sample 2 (A)	14.748	– 0.257	0.748	15.494	– 1.159	0.685	8.550	– 0.054	0.079
Sample 2 (B)	6.686	– 0.146	0.638	7.458	– 0.531	0.800	4.786	0.003	0.001
Sample 3 (A)	5.212	– 0.221	0.722	6.129	– 0.664	0.860	3.054	0.027	0.112
Sample 3 (B)	10.121	– 0.207	0.561	9.225	– 0.341	0.177	6.920	– 0.068	0.092
Sample 5	6.985	– 0.145	0.685	7.833	– 0.587	0.909	4.629	0.048	0.349
Average	---	---	0.699	---	---	0.736	---	---	0.107

	Hyperbolic Model – Eq. 10			Semi-Log Model - Eq. 11			Octahedral Model - Eq. 13′		
	K	n	r^2	K	n	r^2	K′	n′	r^2
Sample 1	0.675	3.676	0.998	3.757	– 0.030	0.441	0.781	0.746	0.781
Sample 2 (A)	11.146	6.961	0.972	4.067	– 0.014	0.536	10.484	1.763	0.953
Sample 2 (B)	1.950	4.641	0.989	3.780	– 0.009	0.411	5.476	0.556	0.787
Sample 3 (A)	2.175	3.075	0.983	3.661	– 0.016	0.492	3.726	0.483	0.739
Sample 3 (B)	8.967	5.356	0.952	3.933	– 0.013	0.513	7.639	1.004	0.745
Sample 5	1.912	4.941	0.993	3.800	– 0.009	0.441	5.460	0.433	0.578
Average	---	---	0.982	---	---	0.472	---	---	0.764

FURTHER DISCUSSIONS

Table 6 lists the basic physical properties and the hyperbolic model constants for the test specimens. Statistical analysis indicated that the hyperbolic model constants (K, n) are both correlated strongly to the moisture content with relative to the OMC (= *w* – OMC), as seen in Figure 6, and moderately correlated to the percent fines (S200) and relative compaction (R). A multi-variable linear regression analysis produced the following outcome for the A-4 soils:

$K = -391.76 + 5.67(PI) + 0.68(S200) - 11.29(w - OMC) + 3.23(R)$ with $r^2 \cong 1.00$

$n = -24.01 + 1.20(PI) - 0.001(S200) - 2.28(w - OMC) + 0.19(R)$ with $r^2 \cong 1.00$

$M_{Ri} = -75.39 + 2.02(PI) + 0.09(S200) - 3.65(w - OMC) + 0.60(R)$ with $r^2 \cong 1.00$

TABLE 6 List of Test Specimen Properties and Hyperbolic Model Constants

	Test Specimen Property Data				Hyperbolic Model	
	PI (%)	S200 (%)	w – OMC (%)	R (%)	K	n
Sample 1	9.9	46.9	+ 1.0	98	0.675	3.676
Sample 2 (A)	9.4	38.2	– 0.7	98	11.146	6.961
Sample 2 (B)	9.4	38.2	+ 0.4	99	1.950	4.641
Sample 3 (A)	8.2	39.3	+ 0.7	102	2.175	3.075
Sample 5	8.0	48.9	– 0.8	95	1.912	4.941

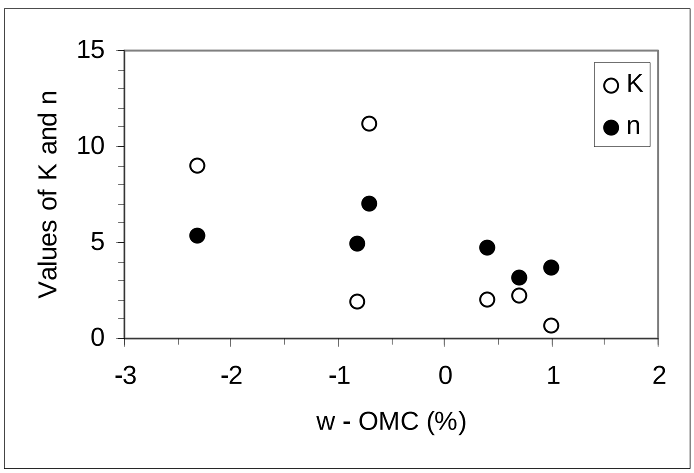

FIGURE 6 Plot of K and n Values vs. (*w* – OMC)

CLOSING REMARKS

Resilient modulus of subgrade soil is one of the key material properties that are required for the mechanistic-empirical design/analysis of multi-layered flexible pavement system. A study was initiated at Ohio University to examine the resilient modulus of fine-grained subgrade soils commonly found in Ohio. The current standard method was first examined to understand the stress conditions the test method induces on the soil specimen. The examination showed the cyclic nature of the stress path and the Octahedral stresses created by the standard test method.

Analysis of the typical RM test results on a A-4 soil specimen confirmed the bilinear relationship between the resilient modulus and the deviatoric stress. This bilinear relationship emerges because the soil specimen is somewhat overconsolidated during the specimen preparation stage (by the static compaction method). The maximum compressive load applied to the soil layers during the specimen compaction closely matched the breakpoint deviatoric stress (σ_{di}) for each test specimen. This shows that the breakpoint deviatoric stress (σ_{di}) is basically the preconsolidation pressure. The analysis also revealed a possible linear relationship between the resilient modulus and the Octahedral stress ratio (σ_{oct}/τ_{oct}) in the logarithmic scale. Five stress models were briefly discussed and evaluated in light of recent laboratory test results. The outcome of the initial study indicated that the hyperbolic model may be the most promising model. This implies that the deviatoric stress alone can adequately express the resilient modulus of the fine-grained soils. It is further noted here that the Octahedral stress model, Eq. 13′, is similar to the universal model recently recommended by AASHTO (Yan and Quintus, 2002):

$$M_R = k_1 p_a \left(\frac{\theta}{p_a} \right)^{k_2} \left(\frac{\tau_{oct}}{p_a} + 1 \right)^{k_3} \qquad (14)$$

The constants in the hyperbolic model appear to be strongly influenced by the basic soil properties. Preliminary correlations were established between the hyperbolic model constants and the test specimens' basic properties (ex. S200, PI, *w*, OMC, γ_d, $\gamma_{d\text{-}max}$, …). Also, the initial relation between the breakpoint resilient modulus and the specimen properties was also obtained. Additional test results will be needed to develop a resilient modulus prediction model for the A-4 Ohio soils at higher statistical confidence levels. Also, the modeling efforts should extend to address the other subgrade soil types (A-6, A-7) commonly found in Ohio.

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