FLEXIBLE PAVEMENT RESPONSE TO ELASTIC MODULUS VARIATION WITH DEPTH

By
Wael Alkasawneh, P.E., Ernie Pan[1], Feng Han, Ronghua Zhu
Dept. of Civil Engineering, The University of Akron, Akron, OH 44325-3905

Roger Green, P.E.
Ohio Department of Transportation, 1980 West Broad Street, Columbus, Ohio 43223

Wael Alkasawneh, Department of Civil Engineering, The University of Akron, Akron, OH 44325-3905, Email: wael@uakron.edu

[1] Corresponding author: Prof. Ernie Pan, Department of Civil Engineering, The University of Akron, Akron, OH 44325-3905, Tel: 330-972-6739 (O), Fax: 330-972-6020, Email: pan2@uakron.edu

Feng Han, Department of Civil Engineering, The University of Akron, Akron, OH 44325-3905, Email: fh7@uakron.edu

Ronghua Zhu, Department of Civil Engineering, The University of Akron, Akron, OH 44325-3905, Email: rz6@uakron.edu

Roger Green, P.E., Ohio Department of Transportation, 1980 West Broad Street, Columbus, Ohio 43223, Tel: 614-995-5993 (O), Fax: 614-995-5998, Email: Roger.Green@dot.state.oh.us

Submission Date: August 30, 2006
Word Count: 3466 words + 8 Figures + 1 Table = **5716** words

ABSTRACT

An innovative method for flexible pavement analysis, where the elastic layer can be subdivided further into many sublayers, was used to study the response under uniform and nonuniform tire pressure distribution. The results showed that the use of uniform tire pressure distribution in flexible pavement analysis can underestimate the pavement response significantly when compared to the response using uniform tire pressure distribution. The effect of modulus variation with depth was further investigated by calculating the pavement response using both the modulus variation and the constant modulus models described in this paper.

KEY WORDS

Multilayered solution, elasticity, flexible pavement, stress discontinuity, strain discontinuity, tire pressure.

INTRODUCTION

Modeling flexible pavement systems has been done using the finite difference method, the finite element method, and the analytical elastic solution. The first two methods could be time consuming and might involve other practical limitations arising from the need to remesh the high stress zones within the pavement and along the tire/pavement interface. The classical elasticity-pavement programs, while being convenient for simple pavement design, cannot model the irregular pressure distribution along the surface and the heat transfer within the pavement, and are limited to at most 20 sublayers (NCHRP, 2004). A recently developed program called ***MultiSmart3D*** (2005) is for the multilayered elastic pavement with unlimited number of layers/sublayers and with the capability of modeling the irregular pressure distributions along the tire/pavement interface. Therefore, the ***MultiSmart3D*** program can be utilized easily to deal with material inhomogeneity in the pavement by varying the modulus with depth using thin elastic sublayers.

FLEXIBLE PAVEMENT

Flexible pavements can be modeled as multilayered elastic systems. Each of the pavement layers can have its own elasticity parameters such as the modulus of elasticity and the Poisson's ratio. The first elastic solution for the layered pavement using the elasticity theory was by Burmister (1943, 1945). For the last half century, the analytical elastic method has been used extensively due to its simplicity with limited number of input parameters.

The use of the average modulus of elasticity for the layer is a common accepted practice among researchers and practitioners due to the difficulty of modeling such variation using conventional elasticity models. Modeling the modulus variation with depth requires a large number of elastic layers where the modulus of elasticity could experience variation with depth due to many factors such as temperature, moisture, etc. This limitation, so far, made the finite element method more appealing. However, the complexity of applying the finite element to pavement engineering made the modeling only a research method rather than a practical tool.

MODULUS VARIATION WITH DEPTH

Material inhomogeneity within the pavement layer/sublayer can be caused by the dependency of the modulus of elasticity on the temperature, moisture, and/or other environmental factors. This fact imposes limitations on the current analytical elastic solution which does not take into consideration the variation of the modulus of elasticity with depth within the same layer. However, our newly developed multilayered program ***MultiSmart3D*** can be applied to any variation of modulus of elasticity in the pavement. The modulus variation within the same layer can be modeled using several sublayers where the sublayer thickness and modulus are different for different sublayers.

STRAIN/STRESS DISCONTINUITY

Displacement, strain, and stress variation with depth in multilayered systems is smooth within the same layer where the modulus of elasticity value either is the same or varies smoothly within the layer. However, the vertical strain ε_{zz} and horizontal stress σ_{xx} can be significantly different on both sides of an interface due to the large variation or jump in the modulus of elasticity in the layers above and below the interface. This discontinuity of strains/stresses can directly and significantly damage the pavement in forms of cracking. Therefore, it is important to consider the strain/stress in any flexible pavement analysis where the modulus varies with depth.

The problem of strain/stress discontinuity at the interface in flexible pavement analysis can be solved by reducing the discontinuity in the modulus of elasticity between the two sides of the interface. In reality, the modulus of elasticity of one layer in the pavement section can be orders higher than that in the subsequent layer. In order to ensure the continuity of the strains/stresses, the variation of the modulus of elasticity with depth should be continuous within the layers and across the interface. The modulus of elasticity variation with depth can be achieved by subdividing each layer into a number of sublayers each with a constant modulus of elasticity. The average modulus of elasticity of all sublayers should be equal to the average modulus of elasticity of the layer.

Field measurement indicates that temperature varies nonlinearly with depth during the day and during the year (Ongel and Harvey, 2004). Therefore, the modulus of elasticity of a layer, in a flexible pavement system, varies with depth due to the dependency of the modulus on the temperature. Pan and Alkasawneh (2006) showed that the variation of the modulus with depth due to temperature variation is nonlinear. Other conditions that should be satisfied by the numerical variation of the modulus with depth include the continuity of the modulus of elasticity at the interfaces of the layer and the equivalence of the average moduli of the elastic layer. Therefore, a linear variation of the modulus with depth cannot be used and a nonlinear variation should be used instead. Using the three conditions above, the variation of the modulus with depth can be achieved using a quadratic equation.

In order to handle the strain/stress "jump" between the two sides of the interface in a multilayered elastic material a simple approach was proposed (Alkasawneh et al., 2006). The method is based on a "controlled" variation of the modulus of elasticity within the elastic layer to ensure the lowest strain/stress "jump". Modulus variation with depth within the same layer is described below (The following steps **a)** through **d)** can be equally applied to other layers as well).

a). Select the thickness and the elasticity parameters (modulus of elasticity and Poisson's ratio) for each main layer in the multilayered elastic system, for example, layer *I* in **Figure 1**.

b). Use the modulus of elasticity in the upper layer (E_{I-1}) and the modulus of the elasticity of the subsequent layer (E_{I+1}) as boundary conditions to control the moduli variation with depth within layer *I*, see **Figure 2**.

c). Use the quadratic equation to describe the variation of the modulus of elasticity with depth in the layer. The resulting system of equations contains two equations and three unknowns (the three unknown constants for the quadratic equation). The needed third equation to solve the linear system of equations can be obtained using:

$$\int_0^{h_I} E(z)dz = E_I h_I \tag{1}$$

where $E(z)$ is the quadratic modulus of elasticity function in layer *I* as a function of depth z and it is equal to $(a + bz + cz^2)$, where *a*, *b*, and *c* are the three unknowns. Equation (1) ensures that the resulting modulus variation within layer *I* will always result in the same average modulus. Therefore, the three simultaneous equations can be solved to find the three unknowns.

d). Subdivide layer *I* into a number of sublayers with the modulus of each sublayer being determined using the quadratic equation at the given depth. The constant modulus of elasticity within each sublayer should ensure a smooth transition of stresses/strains between the two subsequent sublayers (e.g., (j-1) and (j) sublayers). This condition can be satisfied using:

$$E_{j-1} / E_j \approx 0.90 - 1.00 \qquad (2)$$

where E_{j-1} and E_j are the moduli of elasticity of the sublayers (j-1) and (j), respectively. This condition is very important since it will reduce the effect of the modulus variation with depth.

e). Calculate the response of the multilayered elastic system using ***MultiSmart3D*** program.

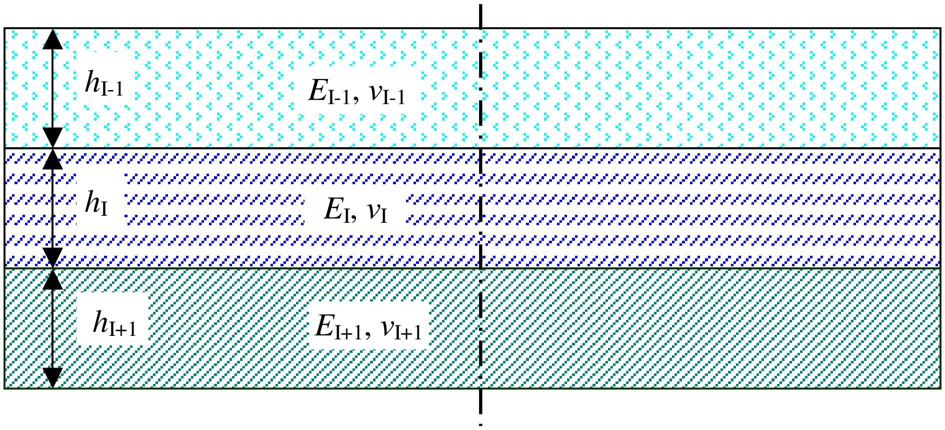

FIGURE 1 A multilayered elastic system.

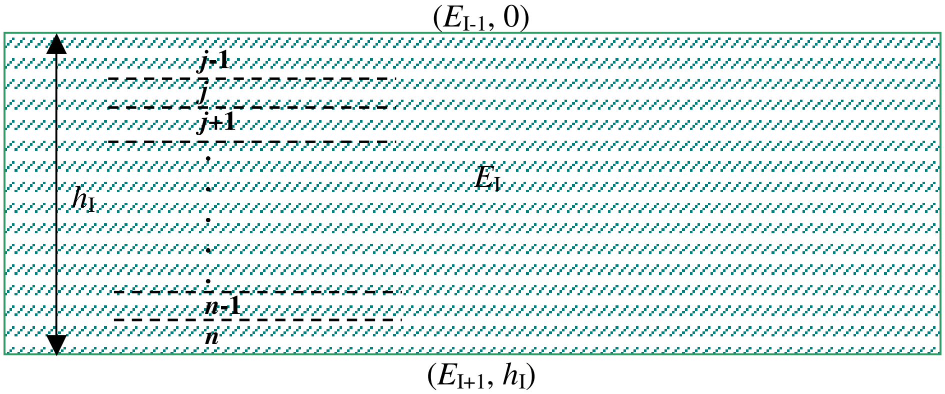

FIGURE 2 Assumed modulus boundary conditions for layer I.

EFFECT OF MODULUS VARIATION WITH DEPTH

The above method was applied to a flexible pavement system to demonstrate the applicability of the new method and to establish some guidelines regarding the use of the new method. The typical flexible pavement section was summarized in **Table 1**. The contact pressure at the surface of the pavement, as shown in **Figure 3**, was measured by Texas Department of Transportation (Luo and Prozzi, 2005). The equivalent pressure taking into consideration the applied load was estimated to be 690 kPa acting on a circle with a diameter of 220.3 mm (Luo and Prozzi, 2005). Pavement responses below the center of the contact pressure area were calculated using the ***MultiSmart3D*** program. The coordinate system is chosen such that the x-

and *y*-axes are on the surface of the pavement (*z*=0) whilst the *z*-axis is vertical to the x-y plane and extends along the depth direction.

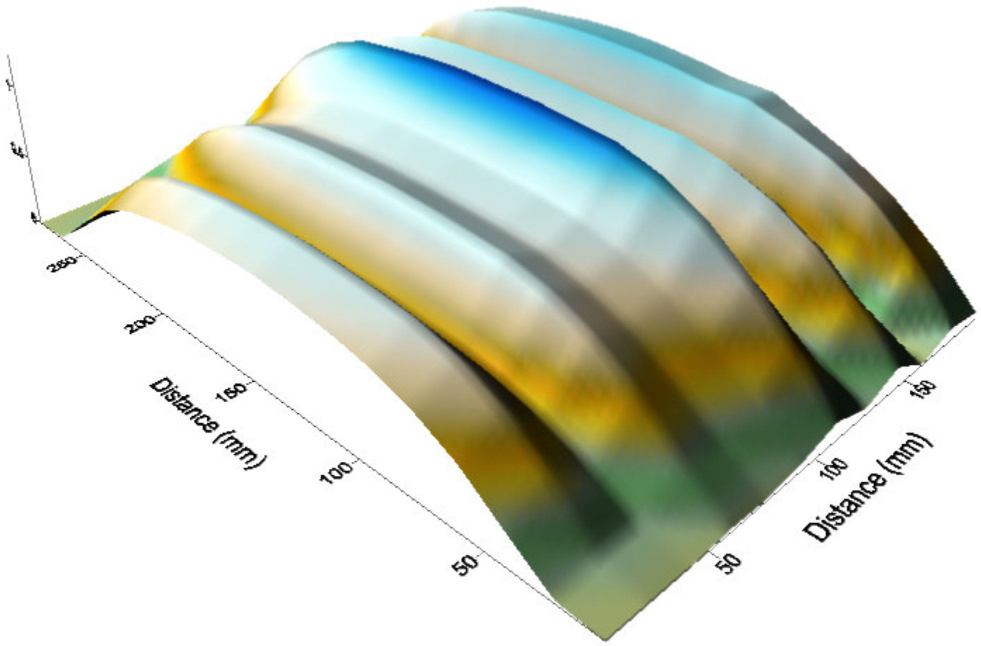

FIGURE 3 Measured nonuniform tire pressure distribution.

TABLE 1 Parameters of a Typical Flexible Pavement Example

Layer	Thickness (cm)	Resilient Modulus (MPa)	Poisson's Ratio
AC Layer	15	3500	0.3
Base Layer	25	700	0.3
Subbase Layer	25	300	0.3
Subgrade Layer	Infinite Half-Space	100	0.3

The interface strain/stress jump between the AC layer and the base layer was studied by the proposed method. The AC layer was subdivided into 125 sublayers. The modulus of elasticity within each sublayer is constant and equal to the average modulus of elasticity of the sublayer. The pavement response within the AC layer was calculated at 120 depth points using both the constant (average) and the quadratic modulus variation models. Pavement response was also calculated at an additional point (a total of 121 points) which was located immediately below the interface between the AC layer and the base layer in order to study the jump of the strain/stress. The effect of nonuniform tire pressure, rather than the equivalent pressure distribution, was also investigated using the modulus variation models: The modulus variation with depth using constant and quadratic modulus variation functions (Alkasawneh et al., 2006). The pavement responses using different models are shown in **Figures 4** through **8**.

Figure 4 shows that the use of the average constant modulus can slightly underestimate the displacement at any point within the AC layer except at the AC/base interface, as compared to the quadratic model with 125 sublayers. The maximum difference of displacements in the AC layer using the quadratic distribution compared to the displacements estimated using the average modulus was approximately 1.5%.

Figure 5 shows the variation of the vertical component of strain (ε_{zz}) with depth. It is clear that the constant modulus of elasticity can overestimate the strains within approximately the top 60% of the AC layer and underestimate the strains in the lower 40% of the AC layer. In addition, the strain jump between the two sides of the AC/base interface can be up to 130% in the constant modulus case while it was less than 2% for the quadratic variation using 125 sublayers. On the other hand, the maximum differences of strains in the upper 60% and the lower 40% of the AC layer between the quadratic and constant models were approximately 40% and 58%, respectively.

Figure 6 shows the variation of the horizontal component of strain (ε_{xx}) with depth. The strains in the upper 30% of the AC layer are almost identical regardless of the modulus variation models within the layer. However, using the constant modulus can underestimate the strains within the lower 70% of the AC layer. The maximum difference of the strains in the lower 70% of the AC layer from the quadratic and constant models is about 40%. Similar behavior can be observed for the other horizontal strain (ε_{yy}) due to symmetry.

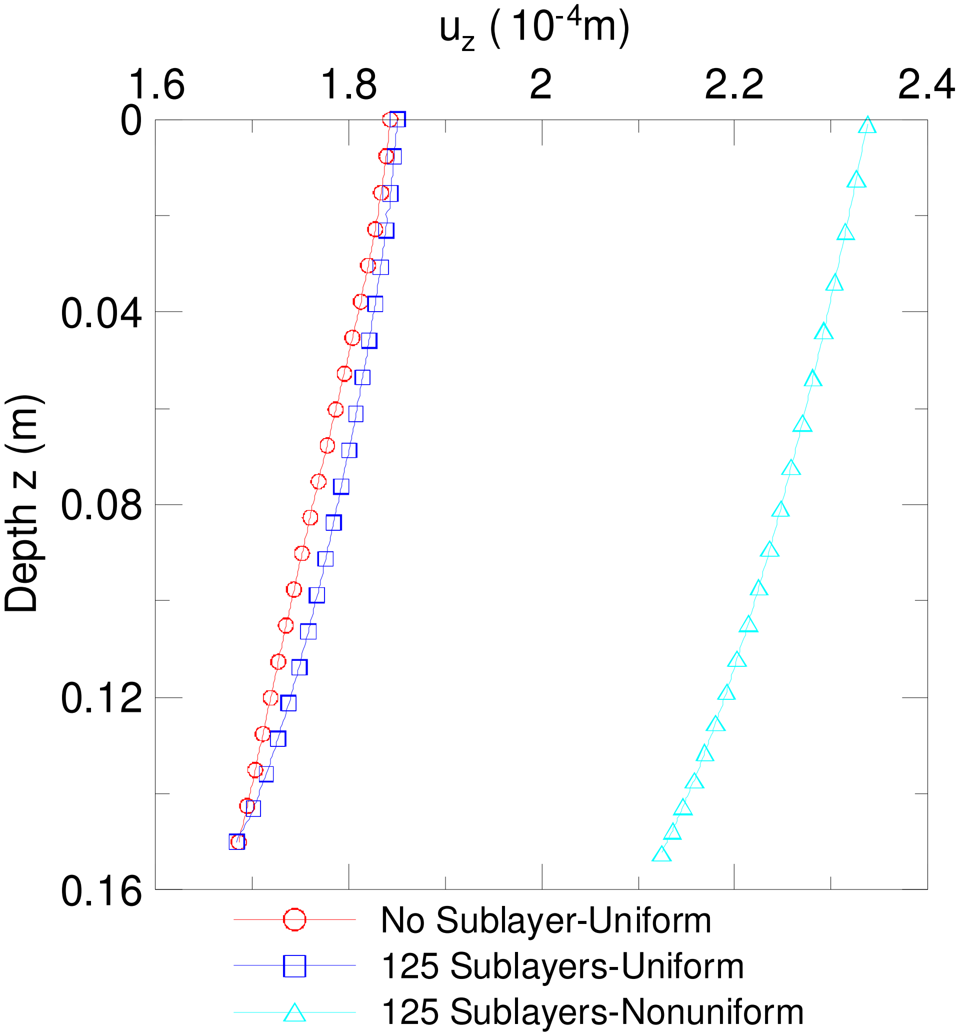

FIGURE 4 Displacement (u_z) variation with depth in the AC layer.

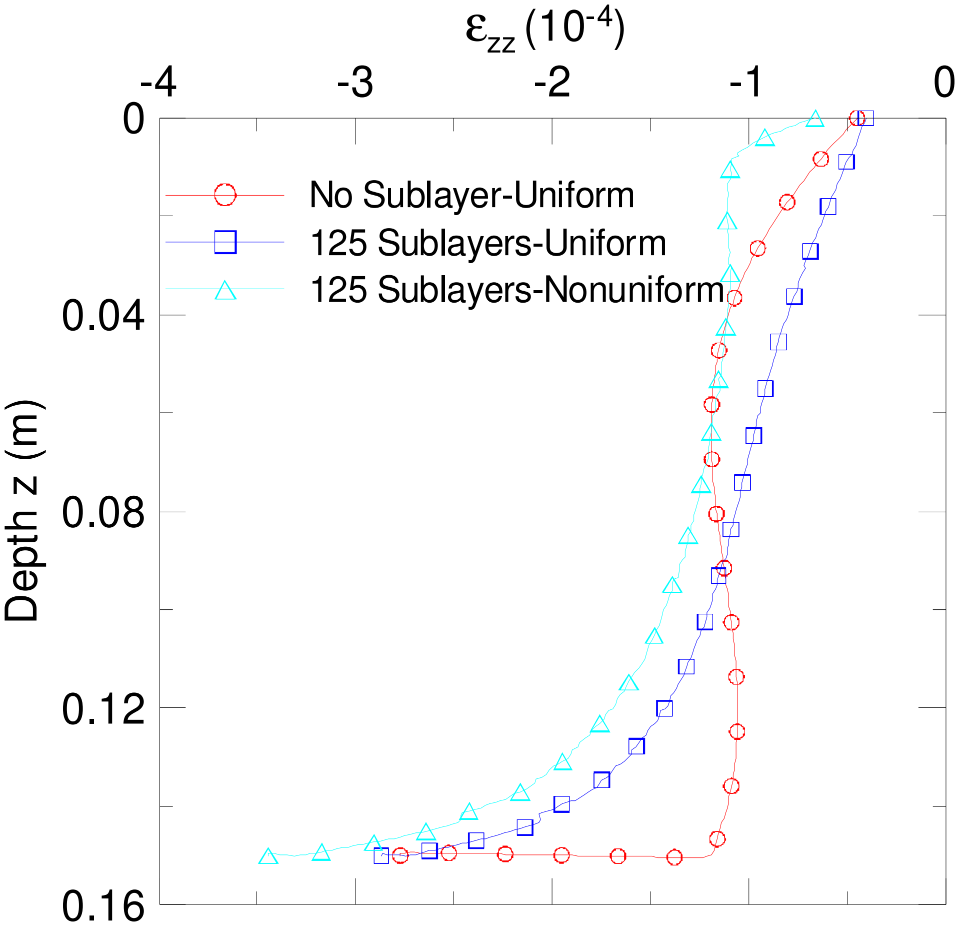

FIGURE 5 Vertical strain (ε_{zz}) variation with depth in the AC layer.

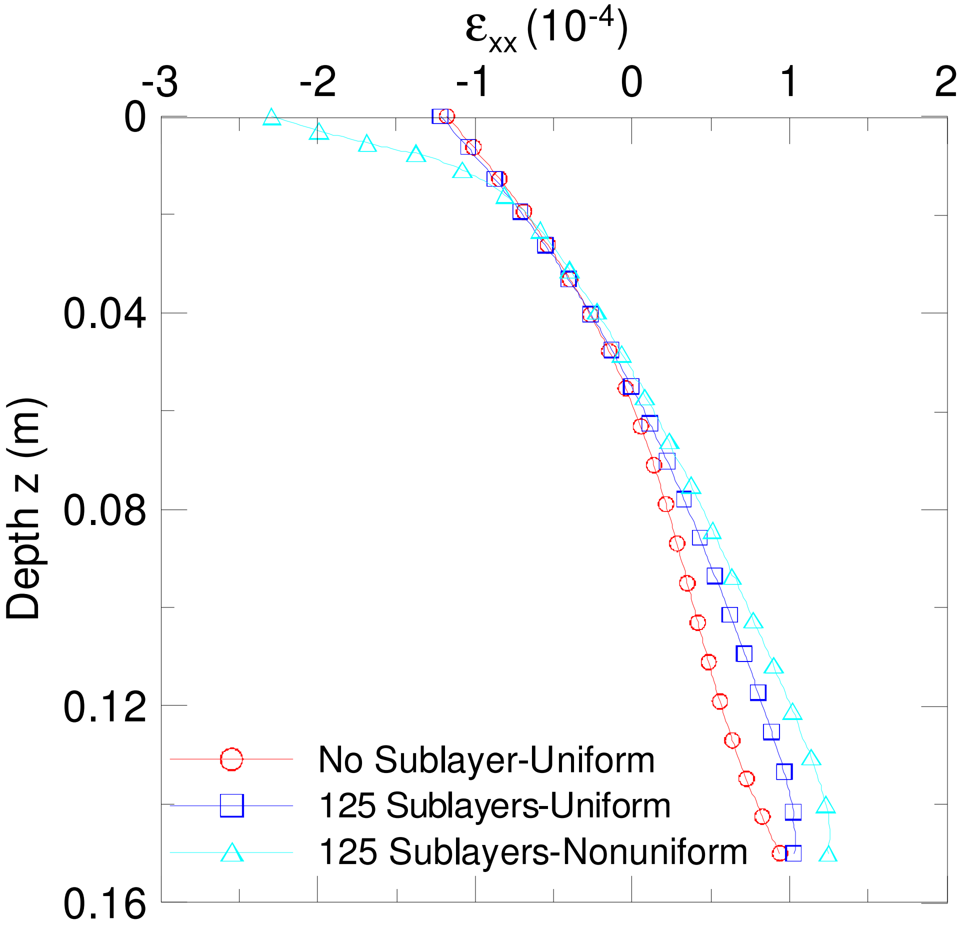

FIGURE 6 Horizontal strain (ε_{xx}) variation with depth in the AC layer.

Figure 7 shows the variation of the vertical component of stress (σ_{zz}) with depth. The difference between the stresses estimated using the quadratic and constant models is negligible in the top 20% and lower 80% of the AC layer, whilst it is less than 10% outside the 20%-80% thickness range.

Figure 8 shows the variation of the horizontal component of stress (σ_{xx}) with depth. The difference between the estimated stresses using the quadratic and constant models is between 3% and 48%, with the maximum difference being at the interface between the AC and base layers.

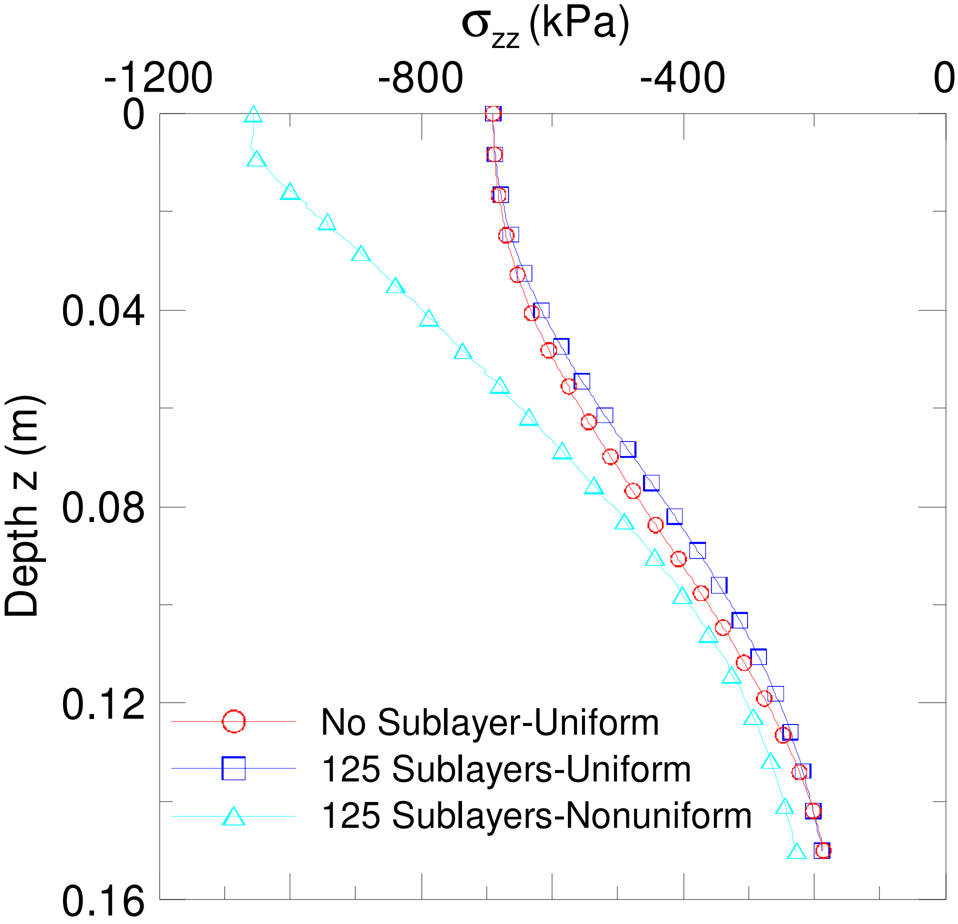

FIGURE 7 Vertical stress (σ_{zz}) variation with depth in the AC layer.

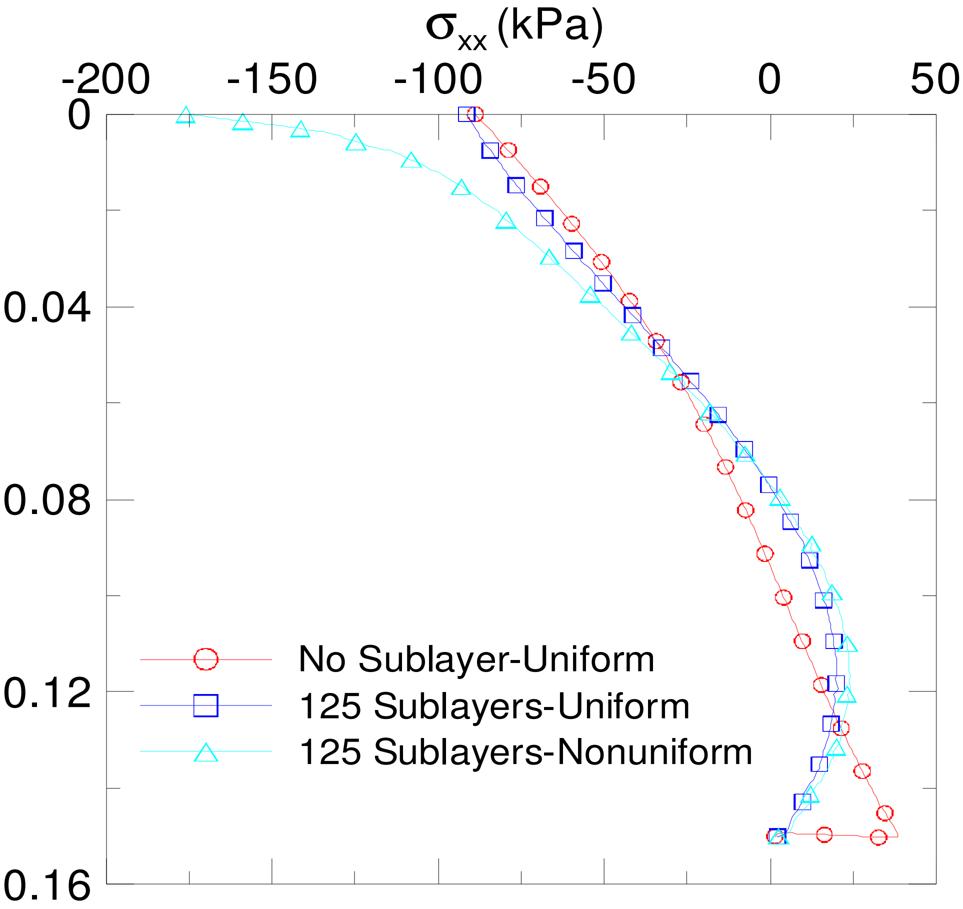

FIGURE 8 Horizontal stress (σ_{xx}) variation with depth in the AC layer.

EFFECT OF TIRE PRESSURE DISTRIBUTION

The measured tire pressure distribution along the surface of the AC layer represents the actual pressure distribution that should be used in any pavement analysis. However, measuring such pressure distribution is not trivial. It is more customary to use uniform tire pressure distribution over a circular area rather than using the actual nonuniform pressure distribution over the footprint of the tire where the geometry of the footprint is controlled by the tire type, speed, tread geometry and configuration, load on tire, and pressure inside tire. The response of the previous flexible pavement example under nonuniform pressure distribution is calculated using the MultiSmart3D program and compared to the results from modulus variation method with unifrom pressure distribution.

It can be seen that the measured tire pressure can result in higher displacement values along the AC profile. The displacement values using the traditional equivalent uniform pressure distribution can lead to misleading displacement predictions. The difference between the calculated displacements using the uniform and nonuniform pressure distributions can be up to 27% regardless of the modulus variation models with depth. In this example, displacements under the nonuniform pressure distribution were higher than those under the uniform pressure distribution below the center of both contact areas. Displacement in the AC layer below the center of the nonuniform pressure area is shown in **Figure 4**.

The main reason for rutting is the compressive strain on the top of the subgrade and therefore predicting the strains correctly can lead to better design and analysis of flexible pavements. Using the nonuniform pressure distribution (with modulus variation with depth) in the analysis produced strains that are 2.2 times the strain from the uniform pressure distribution within the top of the AC layer. The ratio between strains calculated using nonuniform and uniform pressure distribution decreases with depth to approximately 20% between the middle and the bottom of the AC layer. As shown in **Figure 5**, the effect of the nonuniform pressure distribution is significant within the top half of the AC layer.

Fatigue cracking is largely influenced by the horizontal tensile strains at the bottom of the AC layer. Using the nonuniform pressure distribution (with modulus variation with depth) in the analysis produced strains that are 2.1 times the strain from the uniform pressure distribution within the top 10% of the layer. The difference between the horizontal tensile strains from both models reduced below the top 10% of the layer to approximately 30% within the middle of the layer to approximately 20% at the bottom of the AC layer, as shown in **Figure 6**. This difference in strains shows that using the uniform pressure distribution will always underestimate the horizontal strains.

Vertical and horizontal stresses were always underestimated when the uniform pressure distribution is used, as shown in **Figures 7** and **8**, respectively. The vertical stresses at the top of the AC layer using the nonuniform pressure distribution were approximately 1.5 times those calculated using the uniform pressure distribution. The difference between the calculated vertical stresses using nonuniform and uniform pressure distributions reduced to 20% between the middle and the bottom of the AC layer. The horizontal stresses at the top of the AC layer using the nonuniform pressure distribution were approximately 90% higher than those calculated using the uniform pressure distribution. The difference between the calculated horizontal stresses using nonuniform pressure distribution and uniform pressure distribution reduced to 20% between the middle and the bottom of the AC layer.

DISCUSSIONS AND CONCLUSIONS

The average modulus of elasticity is not recommended for the analysis and design of flexible pavements. The continuous variation of the modulus in the AC layer can reduce the “jump” in the stresses and strains between the two sides of the interface. The modulus variation within any layer in the pavement system can be performed using a quadratic relation in which the average elasticity moduli are the boundary conditions. The applicability of this method is demonstrated using the ***MultiSmart3D*** algorithm, recently developed at the University of Akron under the sponsorship of ODOT/FHWA. This new algorithm is superior to any available multilayered flexible pavement program since unlimited number of layers can be used.

The use of the quadratic modulus variation in the pavement also shows that the inhomogeneity of the AC layer can be modeled using the effective multilayered elastic approach. By increasing the number of sublayers, which will in turn reduce the modulus variation with depth, any realistic modulus variation in pavement (due to temperature, moisture, or other environmental factors) can be accurately simulated. However, modeling variation of the resilient modulus using sublayering can be difficult using most of the commercially available programs as most existing multilayered elastic programs limit the number of input layers and the thickness of each layer.

Modeling measured tire pressure distribution is not possible using most of the commercially available multilayer elastic pavement programs, and hence a circular contact area with uniform pressure distribution is commonly used for simplicity instead. The ***MultiSmart3D*** program was used to calculate the pavement response in the AC layer due to nonuniform pressure distribution using the modulus variation model. Results showed that the flexible pavement response was always underestimated using the uniform pressure distribution approach. The response was highly underestimated within the top half of the pavement while it was approximately 20% underestimated in the bottom half of the pavement.

Accurate modeling of the pavement response is very important since inaccurate modeling can either overestimate or underestimate the pavement life significantly. Problems in pavement currently observed may be due to the limitation of modeling the tire pressure distribution. Such limitation is mainly due to the lack of accurate measured tire pressure distribution and the difficulty of modeling such nonuniform distribution using the commercially available multilayer elastic programs. However, the ***MultiSmart3D*** program proved to be very practical and powerful in calculating the flexible pavement response under nonuniform tire pressure distribution.

ACKNOWLEDGEMENTS: The authors are grateful for the support by ODOT/FHWA under grant ODOT 20943.

REFERENCES

Alkasawneh, W., Pan, E., Han, F., Zhu, R., and Green, R. Effective Modulus Variation and Field Distributions with Depth Using a Multilayered Pavement Program, *ASCE J. Transport. Eng.*, 2006 (submitted).

Burmister, D. M. The Theory of Stresses and Displacements in Layered Soil Systems and applications to the Design of Airport Runways, *Proc., Highway Research Board*, Vol. 23, 1943, 126-144.

Burmister, D. M. The General Theory of Stresses and Displacements in layered Soil Systems, *Journal of Applied Physics*, 16, 1945, 84-94, 126- 127,296-302.

Burmister, D. M. Evaluation of Pavement Systems of the WASHO Road Test by Layered Systems Methods, *Highway Research Board Bulletin*, No. 177, 1958, 26-54.

Computational Modeling and Simulation Group. *MultiSmart3D.* Department of Civil Engineering, The University of Akron, Akron, OH, 2005.

Federal Highway Administration (FHWA). *Highway Statistics 2000.* Office of Highway Policy Information, Federal Highway Administration. Washington, D.C, 2000.

Ongel, A. and Harvey, J. *Analysis of 30 Years of Pavement Temperatures using the Enhanced Integrated Climate Model (EICM)*, Pavement Research Center, University of California Davis, 2004.

Huang, Y. H. *Pavement Analysis and Design*, Prince-Hall, Inc, 1993.

Luo, R., and Prozzi, J. A. *Evaluation of the Joint Effect of Wheel Load and Tire Pressure on Pavement Performance*, University of Texas, Research Report SWUTC/05/167245-1, 2005.

National Research Council. *Guide for Mechanistic-Empirical Design*, National Cooperative Highway Research Program (NCHRP), 2004.

Pan, E. and Alkasawneh, W. *An Exploratory Study on Functionally Graded Materials with Application to Multilayered Pavement Design*, Report No. FHWA/OH-R6936-01, Ohio Department of Transportation, Columbus, Ohio, 2006.

Yoder, E. J. *Principles of Pavement Design*, John Wiley & Sons, Inc., London, 1959.